Model Predictive Control Toolbox™

Getting Started Guide

R2014**b**

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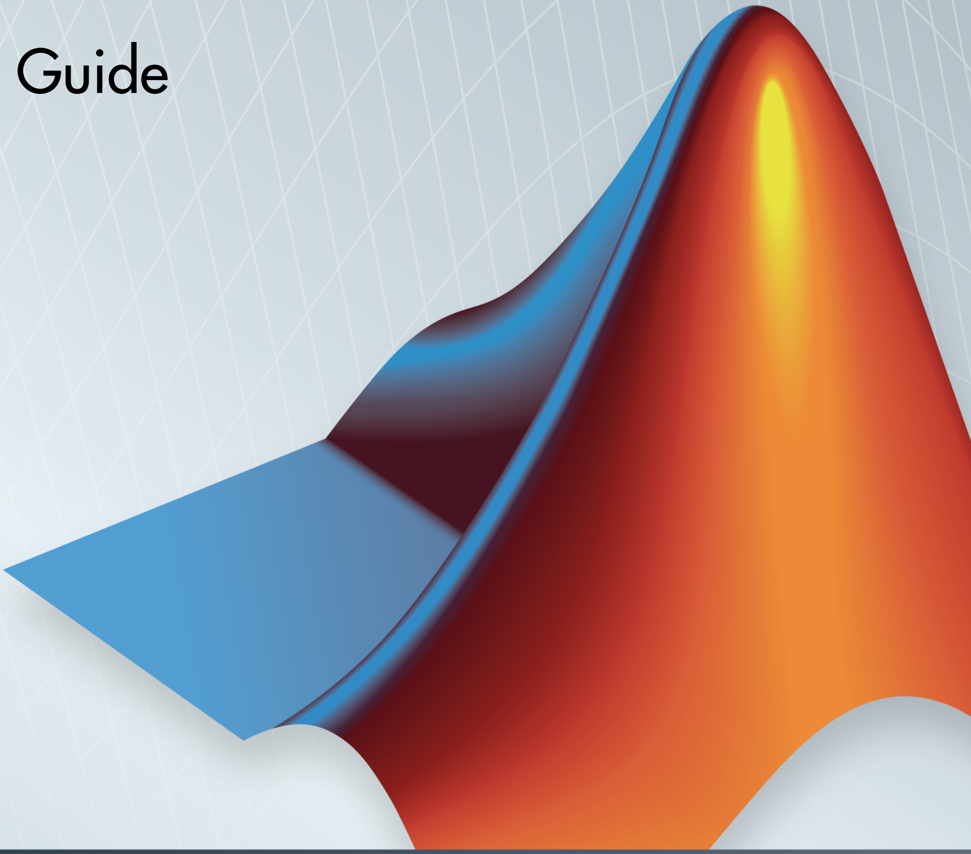

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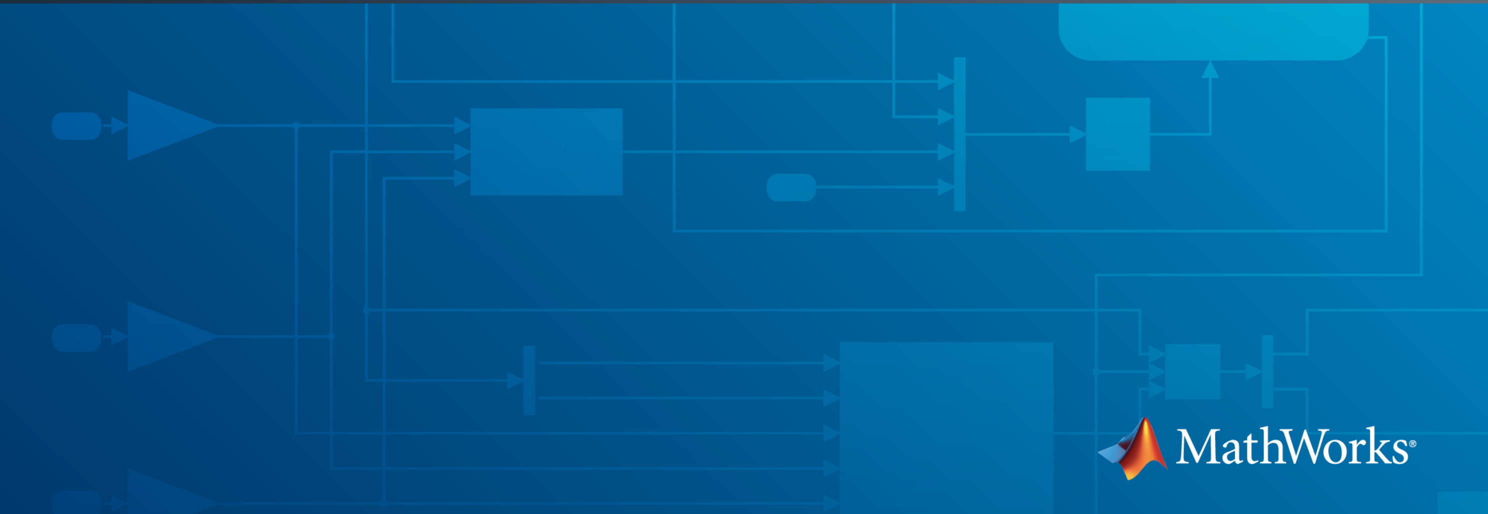

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Model Predictive Control Toolbox™ Getting Started Guide

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Introduction

Model Predictive Control Toolbox Product Description

Design and simulate model predictive controllers

Model Predictive Control Toolbox™ provides functions, an app, and Simulink® blocks for systematically analyzing, designing, and simulating model predictive controllers. You can specify plant and disturbance models, horizons, constraints, and weights. The toolbox enables you to diagnose issues that could lead to run-time failures and provides advice on tuning weights to improve performance and robustness. By running different scenarios in linear and nonlinear simulations, you can evaluate controller performance.

You can adjust controller performance as it runs by tuning weights and varying constraints. You can implement adaptive model predictive controllers by updating the plant model at run time. For applications with fast sample times, you can develop explicit model predictive controllers. For rapid prototyping and embedded system design, the toolbox supports C-code and IEC 61131-3 Structured Text generation.

Key Features

- Design and simulation of model predictive controllers in MATLAB® and Simulink
- Customization of constraints and weights with advisory tools for improved performance and robustness
- Adaptive MPC control through run-time changes to internal plant model
- Explicit MPC control for applications with fast sample times using pre-computed solutions
- Control of plants over a wide range of operating conditions by switching between multiple model predictive controllers
- Specialized model predictive control quadratic programming (QP) solver optimized for speed, efficiency, and robustness
- Support for C-code generation with Simulink Coder™ and IEC 61131-3 Structured Text generation with Simulink PLC Coder™

Using the Documentation

If you have limited experience with MATLAB or Model Predictive Control Toolbox software, read this guide first. It shows how to:

- Define your plant using Control System Toolbox™ modeling tools (LTI transfer function and state space models).
- Derive a linear plant model from a nonlinear Simulink representation.
- Design Model Predictive Control for your plant using `mpctool`, the graphical user interface (GUI), or special commands.
- Simulate Model Predictive Control performance using `mpctool`, Simulink, or commands.

If you have experience with an earlier Model Predictive Control Toolbox release, we advise you to read this document to familiarize yourself with the many new features and the new command syntax (the earlier syntax is still available, but the underlying code is no longer supported).

If you need more details, see the online documentation. To access it from the MATLAB prompt, type:

```
doc
```

When the help dialog box appears, select **Model Predictive Control Toolbox** in the **Contents** pane. This displays a roadmap with links to the available documentation components. Briefly, these are:

- Getting Started. The online version of this document.
- MPC Problem Setup. Mathematical details of the Model Predictive Control Toolbox algorithm and user specifications required for controller design.
- MPC Simulink Library. Describes the Model Predictive Controller Block and its use within Simulink.
- Case-Study Examples. Example toolbox applications.
- The Design Tool. Reference manual for `mpctool`, the GUI.
- Functions. Reference manual describing each Model Predictive Control Toolbox function (used for controller design and simulation in MATLAB commands and scripts).
- Blocks. Reference manual describing the Model Predictive Control Toolbox blocks (used for controller design and simulation in Simulink).

- Object Reference. Details of the controller object that represents a complete controller design.
- Release Notes. Summarizes major features of this release, known limitations, etc.

Related Products

MathWorks provides other products that complement and enhance the Model Predictive Control Toolbox functionality. For more information, see `www.mathworks.com`.

Acknowledgments

MathWorks would like to acknowledge the following contributors to Model Predictive Control Toolbox.

Alberto Bemporad

Professor of Control Systems, IMT Institute for Advanced Studies Lucca, Italy. Research interests include model predictive control, hybrid systems, optimization algorithms, and applications to automotive, aerospace, and energy systems. Fellow of the IEEE®. Author of the Model Predictive Control Simulink library and commands.

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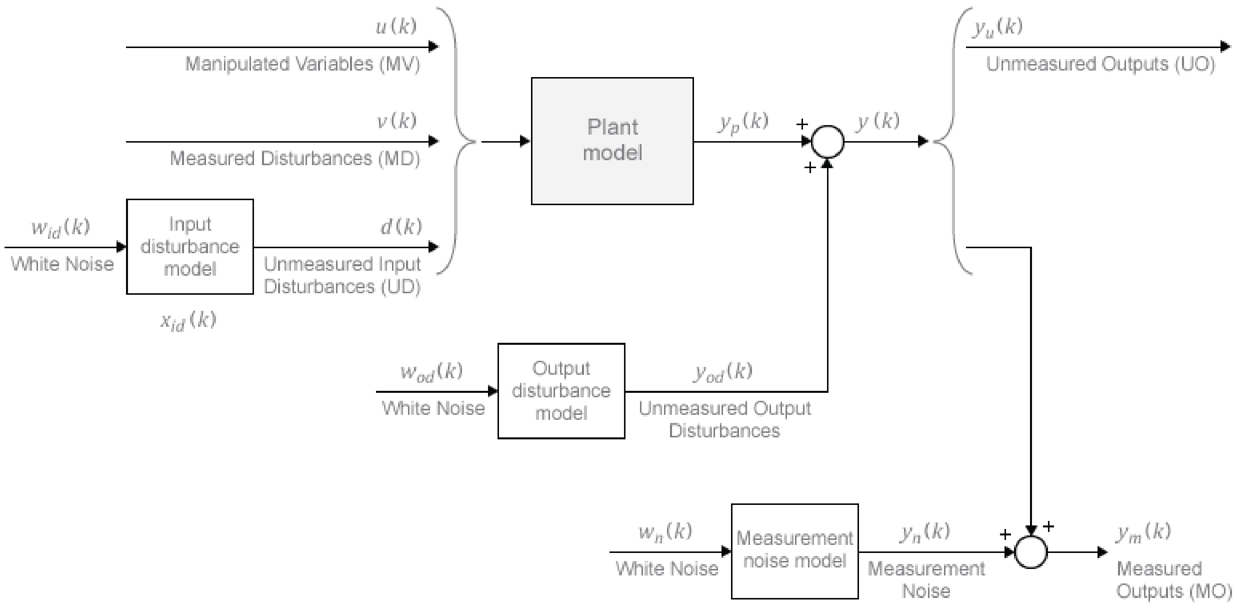
Building Models

MPC Modeling

In this section...

The model structure used in the MPC controller appears in the illustration below. This section explains how the models connect for prediction and state estimation.



Plant Model

The plant model may be in one of the following linear-time-invariant (LTI) formats:

- Numeric LTI models: Transfer function (`tf`), state-space (`ss`), zero-pole-gain (`zpk`).

- Identified models (requires System Identification Toolbox™): `idss`, `idtf`, `idproc`, and `idpoly`.

The MPC controller performs all the estimation and optimization calculations using a discrete-time, delay-free, state-space system, with dimensionless input and output variables. Therefore, when you specify a plant model in the MPC controller, the following model-conversion steps are automatically carried out (if needed):

1. Conversion to state-space. The `ss` command appropriate for the supplied model format generates an LTI state-space model.
2. Discretization or resampling. If the model's sample time differs from the MPC controller's sample time (defined in the `Ts` property), one of the following occurs:
   - If the model is continuous-time, the `c2d` command converts it to a discrete-time LTI object using the controller's sample time.
   - If the model is discrete-time, the `d2d` command resamples it to generate a discrete-time LTI object using the controller's sample time.
3. Delay removal. If the discrete time model includes any input, output, or internal delays, the `absorbDelay` command replaces them with the appropriate number of poles at $z = 0$, increasing the total number of discrete states. The `InputDelay`, `OutputDelay`, and `InternalDelay` properties of the resulting state space model are all zero.
4. Conversion to dimensionless input and output variables. The MPC controller allows you to specify a scale factor for each plant input and output variable. If you do not specify scale factors, they default to unity. The software converts the plant input and output variables to dimensionless form as follows:

$$\begin{aligned} x_p(k+1) &= A_p x_p(k) + B S_i u_p(k) \\ y_p(k) &= S_o^{-1} C x_p(k) + S_o^{-1} D S_i u_p(k). \end{aligned}$$

   Here, where A_p, B, C, and D are constant state space matrices determined in step 3, and:

   - S_i — Diagonal matrix of input scale factors in engineering units.
   - S_o — Diagonal matrix of output scale factors in engineering units.
   - x_p — State vector from step 3 in engineering units. No scaling is performed on state variables.

- u_p — Dimensionless plant input variables.
- u_p — Dimensionless plant output variables.

The resulting plant model has the following equivalent form:

$$x_p(k+1) = A_p x_p(k) + B_{pu}u(k) + B_{pv}v(k) + B_{pd}d(k)$$
$$y_p(k) = C_p x_p(k) + D_{pu}u(k) + D_{pv}v(k) + D_{pd}d(k).$$

Here, $C_p = S_o^{-1}C$, B_{pu}, B_{pv} and B_{pd} are the corresponding columns of BS_i. Also, D_{pu}, B_{pv} and B_{pd} are the corresponding columns of $S_o^{-1}DS_i$. Finally, $u(k)$, $v(k)$ and $d(k)$ are the dimensionless manipulated variables, measured disturbances, and unmeasured input disturbances, respectively.

MPC controller enforces the restriction of $D_{pu} = 0$, which means that MPC controller does not allow direct feedthrough from any manipulated variable to any plant output.

Input Disturbance Model

If your plant model includes unmeasured input disturbances, labeled as $d(k)$ in the diagram of the MPC system, the input disturbance model indicates how $d(k)$ evolves with time. That is, the input disturbance model specifies what type of signal you expect from $d(k)$. If you do not provide an input disturbance model, the controller uses a default input disturbance model. The `getindist` command provides access to the model being used.

The input disturbance model is a key factor influencing the following controller performance attributes:

- Dynamic response to apparent disturbances, i.e., the character of the controller's response when the measured plant output deviates from its predicted trajectory (due to an unknown disturbance or modeling error).
- Asymptotic rejection of sustained disturbances. If the disturbance model predicts a sustained disturbance, controller adjustments continue until the plant output returns to its desired trajectory. This emulates an integral mode in a classical feedback controller.

You can provide the input disturbance model as an LTI state-space (`ss`), transfer function (`tf`), or zero-pole-gain (`zpk`) object. See “Controller State Estimation” for more details about the input disturbance model.

The MPC controller converts the input disturbance model to a discrete-time, delay-free, LTI state-space system using the same steps used to convert the plant model (see “Plant Model” on page 2-2). The result is:

$$\begin{aligned} x_{id}(k+1) &= A_{id}x_{id}(k) + B_{id}w_{id}(k) \\ d(k) &= C_{id}x_{id}(k) + D_{id}w_{id}(k). \end{aligned}$$

Here, A_{id}, B_{id}, C_{id}, and D_{id} are constant state space matrices and:

- $x_{id}(k)$ — $n_{xid} \geq 0$ input disturbance model states.
- $d_k(k)$ — n_d dimensionless unmeasured input disturbances.
- $w_{id}(k)$ — $n_{id} \geq 1$ dimensionless white noise inputs, assumed to have zero mean and unity variance.

Output Disturbance Model

The output disturbance model is a special case of the more general input disturbance model. Its output is directly added to the plant output rather than affecting the plant states. The diagram shows its location in the MPC model hierarchy. The output disturbance model indicates how the disturbance will evolve with time (in other words, what type of disturbance signal you would expect) and it is often used in practice. See “Controller State Estimation” for more details about the output disturbance model.

If you don’t provide an output disturbance model, the MPC controller will create one by default when there is no input disturbance model and augmenting the plant model does not violate state observability.

The `getoutdist` command provides access to the model being used.

Using the same steps as for the plant model (see “Plant Model” on page 2-2), the MPC controller converts the output disturbance model to a discrete-time, delay-free, LTI state-space system. The result is:

$$\begin{aligned} x_{od}(k+1) &= A_{od}x_{od}(k) + B_{od}w_{od}(k) \\ y_{od}(k) &= C_{od}x_{od}(k) + D_{od}w_{od}(k). \end{aligned}$$

Here, A_{od}, B_{od}, C_{od}, and D_{od} are constant state space matrices and:

- $x_{od}(k)$ — $n_{xod} \geq 1$ output disturbance model states.
- $y_{od}(k)$ — n_y dimensionless output disturbances to be added to the dimensionless plant outputs.
- $w_{od}(k)$ — n_{od} dimensionless white noise inputs, assumed to have zero mean, unity variance.

Measurement Noise Model

One of the controller design objectives is to distinguish disturbances, which require a response, from measurement noise, which should be ignored. The measurement noise model has this purpose. The diagram shows its location in the MPC model hierarchy. The measurement noise model indicates how the noise will evolve with time (in other words, what type of noise signal you would expect).

See "Controller State Estimation" for more details about this model model.

Using the same steps as for the plant model (see "Plant Model" on page 2-2), the MPC controller converts the measurement noise model to a discrete-time, delay-free, LTI state-space system. The result is:

$$\begin{aligned} x_n(k+1) &= A_n x_n(k) + B_n w_n(k) \\ y_n(k) &= C_n x_n(k) + D_n w_n(k). \end{aligned}$$

Here, A_n, B_n, C_n, and D_n are constant state space matrices and:

- $x_n(k)$ — $n_{xn} \geq 0$ noise model states.
- $y_n(k)$ — n_{ym} dimensionless noise signals to be added to the dimensionless measured plant outputs.
- $w_n(k)$ — $n_n \geq 1$dimensionless white noise inputs, assumed to have zero mean, unity variance.

If you do not supply a noise model, the default is a unity static gain: $n_{xn} = 0$, D_n is an n_{ym}-by-n_{ym} identity matrix, and A_n, B_n, and C_n are empty.

For an `mpc` controller object `MPCobj`, the property `MPCobj.Model.Noise` provides access to the measurement noise model.

Note: If the minimum eigenvalue of $D_n D_n^T$ is less than 1x10^{-8}, the MPC controller adds 1x10^{-4} to each diagonal element of D_n. This adjustment makes it more likely that the default Kalman gain calculation will succeed.

More About

- "Controller State Estimation"

Signal Types

Inputs

The *plant inputs* are the independent variables affecting the plant. As shown in “MPC Modeling” on page 2-2, there are three types:

Measured disturbances

The controller can't adjust them, but uses them for feedforward compensation.

Manipulated variables

The controller adjusts these in order to achieve its goals.

Unmeasured disturbances

These are independent inputs of which the controller has no direct knowledge, and for which it must compensate.

Outputs

The *plant outputs* are the dependent variables (outcomes) you wish to control or monitor. As shown in “MPC Modeling” on page 2-2, there are two types:

Measured outputs

The controller uses these to estimate unmeasured quantities and as feedback on the success of its adjustments.

Unmeasured outputs

The controller estimates these based on available measurements and the plant model. The controller can also hold unmeasured outputs at setpoints or within constraint boundaries.

You must specify the input and output types when designing the controller. See “Input and Output Types” on page 2-14 for more details.

More About

- “MPC Modeling” on page 2-2

Construct Linear, Time Invariant (LTI) Models

In this section...

Model Predictive Control Toolbox software supports the same LTI model formats as does Control System Toolbox software. You can use whichever is most convenient for your application. It's also easy to convert from one format to another.

The following topics describe the three model formats and the commands used to construct them:

- "Transfer Function Models" on page 2-9
- "Zero/Pole/Gain Models" on page 2-10
- "State-Space Models" on page 2-10
- "LTI Object Properties" on page 2-12
- "Specify Multi-Input Multi-Output (MIMO) Plants" on page 2-16
- "LTI Model Characteristics" on page 2-15

For more details, see the Control System Toolbox documentation.

Transfer Function Models

A transfer function (TF) relates a particular input/output pair. For example, if $u(t)$ is a plant input and $y(t)$ is an output, the transfer function relating them might be:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{s+2}{s^2+s+10}e^{-1.5s}$$

This TF consists of a *numerator* polynomial, s+2, a *denominator* polynomial, s^2+s+10, and a delay, which is 1.5 time units here. You can define G using Control System Toolbox `tf` function:

```
Gtf1 = tf([1 2], [1 1 10], 'OutputDelay', 1.5)
```

Control System Toolbox software builds and displays it as follows:

```
Transfer function:
                 s + 2
exp(-1.5*s) * ------------
              s^2 + s + 10
```

Zero/Pole/Gain Models

Like the TF, the zero/pole/gain (ZPK) format relates an input/output pair. The difference is that the ZPK numerator and denominator polynomials are factored, as in

$$G(s) = 2.5\frac{s+0.45}{(s+0.3)(s+0.1+0.7i)(s+0.1-0.7i)}$$

(zeros and/or poles are complex numbers in general).

You define the ZPK model by specifying the zero(s), pole(s), and gain as in

```
Gzpk1 = zpk( -0.45, [-0.3, -0.1+0.7*i, -0.1-0.7*i], 2.5)
```

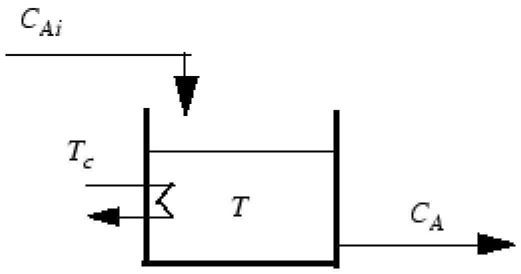
State-Space Models

The state-space format is convenient if your model is a set of LTI differential and algebraic equations. For example, consider the following linearized model of a continuous stirred-tank reactor (CSTR) involving an exothermic (heat-generating) reaction [9]

$$\frac{dC'_A}{dt} = a_{11}C'_A + a_{12}T' + b_{11}T'_c + b_{12}C'_{Ai}$$

$$\frac{dT'}{dt} = a_{21}C'_A + a_{22}T' + b_{21}T'_c + b_{22}C'_{Ai}$$

where C_A is the concentration of a key reactant, T is the temperature in the reactor, T_c is the coolant temperature, C_{Ai} is the reactant concentration in the reactor feed, and a_{ij} and b_{ij} are constants. See the process schematic in CSTR Schematic. The primes (e.g., C'_A) denote a deviation from the nominal steady-state condition at which the model has been linearized.



CSTR Schematic

Measurement of reactant concentrations is often difficult, if not impossible. Let us assume that T is a measured output, C_A is an unmeasured output, T_c is a manipulated variable, and C_{Ai} is an unmeasured disturbance.

The model fits the general state-space format

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where

$$x = \begin{bmatrix} C'_A \\ T' \end{bmatrix} \quad u = \begin{bmatrix} T'_c \\ C'_{Ai} \end{bmatrix} \quad y = \begin{bmatrix} T' \\ C'_A \end{bmatrix},$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The following code shows how to define such a model for some specific values of the a_{ij} and b_{ij} constants:

```
A = [-0.0285  -0.0014
     -0.0371  -0.1476];
B = [-0.0850   0.0238
      0.0802   0.4462];
```

```
C = [0 1
     1 0];
D = zeros(2,2);
CSTR = ss(A,B,C,D);
```

This defines a *continuous-time* state-space model. If you do not specify a sampling period, a default sampling value of zero applies. You can also specify discrete-time state-space models. You can specify delays in both continuous-time and discrete-time models.

Note In the CSTR example, the D matrix is zero and the output does not instantly respond to change in the input. The Model Predictive Control Toolbox software prohibits direct (instantaneous) feedthrough from a manipulated variable to an output. For example, the CSTR model could include direct feedthrough from the unmeasured disturbance, C_{Ai}, to either C_A or T but direct feedthrough from T_c to either output would violate this restriction. If the model had direct feedthrough from T_c, you can add a small delay at this input to circumvent the problem.

LTI Object Properties

The `ss` function in the last line of the above code creates a state space model, `CSTR`, which is an *LTI object*. The `tf` and `zpk` commands described in "Transfer Function Models" on page 2-9 and "Zero/Pole/Gain Models" on page 2-10 also create LTI objects. Such objects contain the model parameters as well as optional properties.

LTI Properties for the CSTR Example

The following code sets some of the `CSTR` model's optional properties:

```
CSTR.InputName = {'T_c', 'C_A_i'};
CSTR.OutputName = {'T', 'C_A'};
CSTR.StateName = {'C_A', 'T'};
CSTR.InputGroup.MV = 1;
CSTR.InputGroup.UD = 2;
CSTR.OutputGroup.MO = 1;
CSTR.OutputGroup.UO = 2;
CSTR
```

The first three lines specify labels for the input, output and state variables. The next four specify the signal type for each input and output. The designations `MV`, `UD`, `MO`, and `UO` mean *manipulated variable*, *unmeasured disturbance*, *measured output*, and *unmeasured* *output*. (See “Signal Types” on page 2-8 for definitions.) For example, the code specifies that input 2 of model `CSTR` is an unmeasured disturbance. The last line causes the LTI object to be displayed, generating the following lines in the MATLAB Command Window:

```
a =
             C_A         T
   C_A  -0.0285  -0.0014
   T    -0.0371  -0.1476

b =
            T_c    C_Ai
   C_A  -0.085  0.0238
   T    0.0802  0.4462

c =
        C_A    T
   T      0    1
   C_A    1    0


d =
         T_c  C_Ai
   T       0     0
   C_A     0     0

Input groups:
    Name    Channels
     MV        1
     UD        2

Output groups:
    Name    Channels
     MO        1
     UO        2

Continuous-time model
```

Input and Output Names

The optional `InputName` and `OutputName` properties affect the model displays, as in the above example. The toolbox also uses the `InputName` and `OutputName` properties to label plots and tables. In that context, the underscore character causes the next character to be displayed as a subscript.

Input and Output Types

General Case

As mentioned in “Signal Types” on page 2-8, Model Predictive Control Toolbox software supports three input types and two output types. In a Model Predictive Control Toolbox design, designation of the input and output types determines the controller dimensions and has other important consequences.

For example, suppose your plant structure were as follows:

Plant Inputs	Plant Outputs
Two manipulated variables (MVs)	Three measured outputs (MOs)
One measured disturbance (MD)	Two unmeasured outputs (UOs)
Two unmeasured disturbances (UDs)	

The resulting controller has four inputs (the three MOs and the MD) and two outputs (the MVs). It includes feedforward compensation for the measured disturbance, and assumes that you wanted to include the unmeasured disturbances and outputs as part of the regulator design.

If you didn't want a particular signal to be treated as one of the above types, you could do one of the following:

- Eliminate the signal before using the model in controller design.
- For an output, designate it as unmeasured, then set its weight to zero (see “Output Weights”).
- For an input, designate it as an unmeasured disturbance, then define a custom state estimator that ignores the input (see “Disturbance Modeling and Estimation”).

Note By default, the toolbox assumes that unspecified plant inputs are manipulated variables, and unspecified outputs are measured. Thus, if you didn't specify signal types in the above example, the controller would have four inputs (assuming all plant outputs were measured) and five outputs (assuming all plant inputs were manipulated variables).

For model `CSTR`, default Model Predictive Control Toolbox assumptions are incorrect. You must set its `InputGroup` and `OutputGroup` properties, as illustrated in the above code, or modify the default settings when you load the model into the design tool.

Use `setmpcsignals` to make type definition. For example

```
CSTR = setmpcsignals(CSTR, 'UD', 2, 'UO', 2);
```

sets `InputGroup` and `OutputGroup` to the same values as in the previous example. The `CSTR` display would then include the following lines:

```
Input groups:
       Name        Channels
    Unmeasured        2
    Manipulated       1



Output groups:
       Name       Channels
    Unmeasured       2
     Measured        1
```

Notice that `setmpcsignals` sets unspecified inputs to `Manipulated` and unspecified outputs to `Measured`.

LTI Model Characteristics

Control System Toolbox software provides functions for analyzing LTI models. Some of the more commonly used are listed below. Type the example code at the MATLAB prompt to see how they work for the `CSTR` example.

Example	Intended Result
`dcgain(CSTR)`	Calculate gain matrix for the `CSTR` model's input/output pairs.
`impulse(CSTR)`	Graph `CSTR` model's unit-impulse response.
`ltiview(CSTR)`	Open the LTI Viewer with the `CSTR` model loaded. You can then display model characteristics by making menu selections.
`pole(CSTR)`	Calculate `CSTR` model's poles (to check stability, etc.).
`step(CSTR)`	Graph `CSTR` model's unit-step response.
`zero(CSTR)`	Compute `CSTR` model's transmission zeros.

Specify Multi-Input Multi-Output (MIMO) Plants

Most Model Predictive Control Toolbox applications involve plants having multiple inputs and outputs. You can use `ss`, `tf`, and `zpk` to represent a MIMO plant model. For example, consider the following model of a distillation column [11], which has been used in many advanced control studies:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21.0s+1} & \dfrac{3.8e^{-8.1s}}{14.9s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} & \dfrac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Outputs y_1 and y_2 represent measured product purities. The control objective is to hold each at specified setpoints. To do so, the controller manipulates inputs u_1 and u_2, the flow rates of reflux and reboiler steam, respectively. Input u_3 is a measured feed flow rate disturbance.

The model consists of six transfer functions, one for each input/output pair. Each transfer function is the first-order-plus-delay form often used by process control engineers.

The following code shows how to define the distillation column model for use in the toolbox:

```
g11 = tf( 12.8, [16.7 1], 'IOdelay', 1.0,'TimeUnit','minutes');
g12 = tf(-18.9, [21.0 1], 'IOdelay', 3.0,'TimeUnit','minutes');
g13 = tf(  3.8, [14.9 1], 'IOdelay', 8.1,'TimeUnit','minutes');
g21 = tf(  6.6, [10.9 1], 'IOdelay', 7.0,'TimeUnit','minutes');
g22 = tf(-19.4, [14.4 1], 'IOdelay', 3.0,'TimeUnit','minutes');
g23 = tf(  4.9, [13.2 1], 'IOdelay', 3.4,'TimeUnit','minutes');
DC = [g11 g12 g13
      g21 g22 g23];
DC.InputName = {'Reflux Rate', 'Steam Rate', 'Feed Rate'};
DC.OutputName = {'Distillate Purity', 'Bottoms Purity'};
DC = setmpcsignals(DC, 'MD', 3)
```

```
-->Assuming unspecified input signals are manipulated variables.

DC =

  From input "Reflux Rate" to output...
                                    12.8
```

```
   Distillate Purity:  exp(-1*s) * ----------
                                   16.7 s + 1

                                   6.6
   Bottoms Purity:  exp(-7*s) * ----------
                                10.9 s + 1

  From input "Steam Rate" to output...
                                    -18.9
   Distillate Purity:  exp(-3*s) * --------
                                   21 s + 1

                                  -19.4
   Bottoms Purity:  exp(-3*s) * ----------
                                14.4 s + 1

  From input "Feed Rate" to output...
                                        3.8
   Distillate Purity:  exp(-8.1*s) * ----------
                                     14.9 s + 1

                                     4.9
   Bottoms Purity:  exp(-3.4*s) * ----------
                                  13.2 s + 1

Input groups:
       Name        Channels
     Measured         3
    Manipulated      1,2

Output groups:
      Name      Channels
    Measured      1,2

Continuous-time transfer function.
```

The code defines the individual transfer functions, and then forms a matrix in which each row contains the transfer functions for a particular output, and each column corresponds to a particular input. The code also sets the signal names and designates the third input as a measured disturbance.

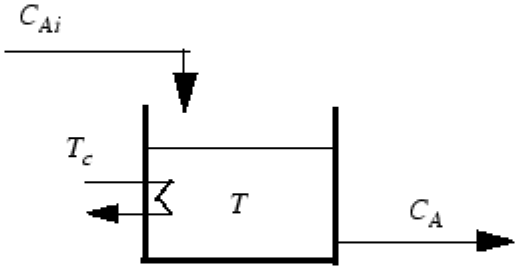
CSTR Model

The linearized model of a continuous stirred-tank reactor (CSTR) involving an exothermic (heat-generating) reaction is represented by the following differential equations:[9]

$$\frac{dC'_A}{dt} = a_{11}C'_A + a_{12}T' + b_{11}T'_c + b_{12}C'_{Ai}$$

$$\frac{dT'}{dt} = a_{21}C'_A + a_{22}T' + b_{21}T'_c + b_{22}C'_{Ai}$$

where C_A is the concentration of a key reactant, T is the temperature in the reactor, T_c is the coolant temperature, C_{Ai} is the reactant concentration in the reactor feed, and a_{ij} and b_{ij} are constants.The primes (e.g., C'_A) denote a deviation from the nominal steady-state condition at which the model has been linearized.



Measurement of reactant concentrations is often difficult, if not impossible. Let us assume that T is a measured output, C_A is an unmeasured output, T_c is a manipulated variable, and C_{Ai} is an unmeasured disturbance.

The model fits the general state-space format

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where

$$x = \begin{bmatrix} C'_A \\ T' \end{bmatrix} \quad u = \begin{bmatrix} T'_c \\ C'_{Ai} \end{bmatrix} \quad y = \begin{bmatrix} T' \\ C'_A \end{bmatrix},$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The following code shows how to define such a model for some specific values of the a_{ij} and b_{ij} constants:

```
A = [-0.0285  -0.0014
     -0.0371  -0.1476];
B = [-0.0850   0.0238
      0.0802   0.4462];
C = [0 1
     1 0];
D = zeros(2,2);
CSTR = ss(A,B,C,D);
```

The following code sets some of the `CSTR` model's optional properties:

```
CSTR.InputName = {'T_c', 'C_A_i'};
CSTR.OutputName = {'T', 'C_A'};
CSTR.StateName = {'C_A', 'T'};
CSTR.InputGroup.MV = 1;
CSTR.InputGroup.UD = 2;
CSTR.OutputGroup.MO = 1;
CSTR.OutputGroup.UO = 2;
```

To view the properties of `CSTR`, enter:

```
CSTR
```

Linearize Simulink Models

Generally, real systems are nonlinear. To design an MPC controller of a nonlinear system, you must model the plant in Simulink.

Although an MPC controller can regulate a nonlinear plant, the model used within the controller must be linear. In other words, the controller employs a linear approximation of the nonlinear plant. The accuracy of this approximation significantly affects controller performance.

This raises the question: how to obtain the approximation? The usual approach is to *linearize* the nonlinear plant at a specified *operating point*. The Simulink environment provides two ways to accomplish this:

- "Linearization Using MATLAB Code" on page 2-20
- "Linearization Using Linear Analysis Tool in Simulink Control Design" on page 2-22

Note: Simulink Control Design™ software must be installed to linearize nonlinear Simulink models.

Linearization Using MATLAB Code

This example shows how to programmatically obtain a linear model of a plant.

For this example, linearize the CSTR model, `CSTR_OpenLoop`. The model inputs are the coolant temperature (manipulated variable of the MPC controller), limiting reactant concentration in the feed stream, and feed temperature. The model states are the temperature and concentration of the limiting reactant in the product stream. You measure both states and use them for feedback control.

1 Obtain a steady-state operating point.

   The operating point defines the nominal conditions at which you will linearize. It is usually a steady state condition.

   Suppose you plan to operate the CSTR with the output concentration at 2 kmol/m^3. The nominal feed concentration is 10 kmol/m^3 and the nominal feed temperature is 300 K. Create an operating point specification object to define the steady state.

```
opspec = operspec('CSTR_OpenLoop');
opspec = addoutputspec(opspec,'CSTR_OpenLoop/CSTR',2);
opspec.Outputs(1).Known = true;
opspec.Outputs(1).y = 2;

op1 = findop('CSTR_OpenLoop',opspec);
```

The calculated operating point is `C_A = 2 kmol/m`^{3} and `T_K = 373 K`. Notice that the coolant temperature at steady state is also given as 299 K, which is the nominal value of the manipulated variable of the model predictive controller.

To specify:

- Values of known inputs, use the `Input.Known` and `Input.u` fields of `opspec`
- Initial guesses for state values, use the `State.x` field of `opspec`

For example, the following code specifies the coolant temperature as 305K and initial guess values of the `C_A` and `T_K` states before calculating the steady-state operating point:

```
opspec = operspec('CSTR_OpenLoop');
opspec.States(1).x = 1;
opspec.States(2).x = 400;
opspec.Inputs(1).Known = true;
opspec.Inputs(1).u = 305;

op2 = findop('CSTR_OpenLoop',opspec);
```

2 Specify linearization inputs/outputs.

If the linearization input /output signals are already defined in the model, as in `CSTR_OpenLoop`, then use the following to programmatically obtain the signal set:

```
io = getlinio('CSTR_OpenLoop');
```

Otherwise, specify the input/output signals as shown:

```
io(1) = linio('CSTR_OpenLoop/Feed Concentration', 1, 'input');
io(2) = linio('CSTR_OpenLoop/Feed Temperature',   1, 'input');
io(3) = linio('CSTR_OpenLoop/Coolant Temperature', 1, 'input');
io(4) = linio('CSTR_OpenLoop/CSTR', 1, 'output');
io(5) = linio('CSTR_OpenLoop/CSTR', 2, 'output');
```

3 Linearize the model at the specified operating point.

```
sys = linearize('CSTR_OpenLoop', op1, io)

sys

a =
              c_a      t_k
   c_a         -5  -0.3427
   t_k      47.68    2.785

b =
        coolant temp  feed concent  feed tempera
   c_a             0             1             0
   t_k           0.3             0             1

c =
             c_a  t_k
   cstr/1     0    1
   cstr/2     1    0

d =
           coolant temp  feed concent  feed tempera
   cstr/1             0             0             0


   cstr/2             0             0             0
```

`sys` is a continuous-time state-space model.

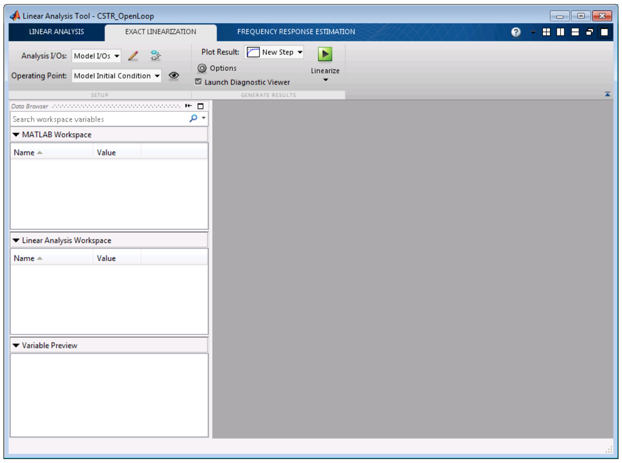
Linearization Using Linear Analysis Tool in Simulink Control Design

This example shows how to linearize a Simulink model using the Linear Analysis Tool. The Linear Analysis Tool, provided by Simulink Control Design, is a graphical interface for model linearization.

1. Open the Simulink model.

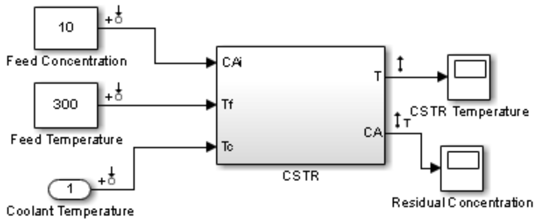
   ```
   sys = 'CSTR_OpenLoop';
   open_system(sys)
   ```
2. Open the Linear Analysis Tool for the model.

   In the Simulink model window, select **Analysis** > **Control Design** > **Linear Analysis**.


Linear Analysis Tool - CSTR_OpenLoop
LINEAR ANALYSIS
EXACT LINEARIZATION
FREQUENCY RESPONSE ESTIMATION
Analysis I/Os: Model I/Os
Operating Point: Model Initial Condition
Plot Result: New Step
Options
Launch Diagnostic Viewer
Linearize
SETUP
GENERATE RESULTS
Data Browser
Search workspace variables
MATLAB Workspace
Name
Value
Linear Analysis Workspace
Name
Value
Variable Preview

3 Specify linearization input/output signals.

- In the Simulink model window, right-click the output signal from the `Feed Concentration` block. Select **Linear Analysis Points** > **Input Perturbation**.
- In the Simulink model window, right-click the output signal from the `Feed Temperature` block. Select **Linear Analysis Points** > **Input Perturbation**.
- In the Simulink model window, right-click the output signal from the `Coolant Temperature` block. Select **Linear Analysis Points** > **Input Perturbation**.
- In the Simulink model window, right-click the input signal to the `CSTR Temperature` block. Select **Linear Analysis Points** > **Output Measurement**.
- In the Simulink model window, right-click the input signal to the `Residual Concentration` block. Select **Linear Analysis Points** > **Output Measurement**.



4 Specify the residual concentration as a trim constraint.

In the Simulink model window, right-click the `CA` output signal from the `CSTR` block. Select **Linear Analysis Points** > **Trim Output Constraint**.

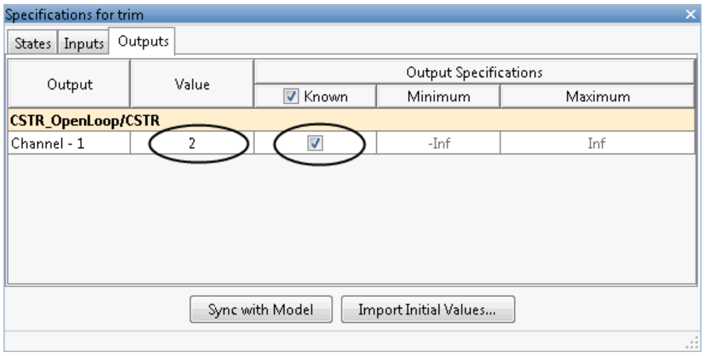
5 Opens the **Specifications for trim** dialog box.

In the Linear Analysis Tool, click the **Exact Linearization** tab. In the **Operating Point** list, select `Trim model`. The **Trim Model** tab opens. Click **Specifications**.

6 Specify the residual concentration as a known output.

Select the **Outputs** tab and set the value of the **CSTR_OpenLoop/CSTR** (`CA`) to 2 kmol/m^3.

Select the **Known** check box.

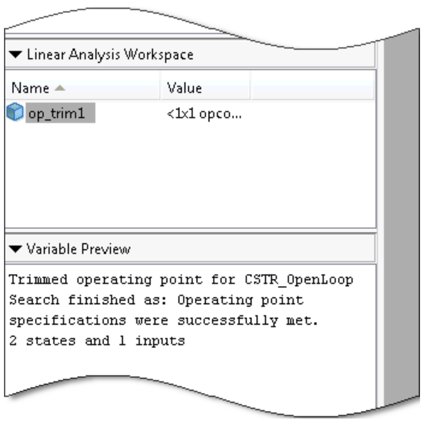


Output	Value	Output Specifications		
		✓ Known	Minimum	Maximum
CSTR_OpenLoop/CSTR				
Channel - 1	2	✓	-Inf	Inf

7 Create and verify the operating point.

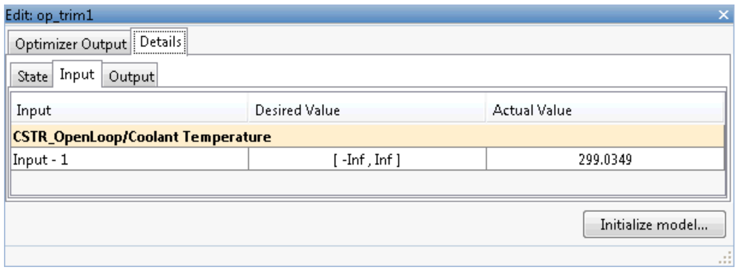
In the **Trim Model** tab, click **Trim**.

The operating point `op_trim1` appears in the **Linear Analysis Workspace**.



```
Trimmed operating point for CSTR_OpenLoop
Search finished as: Operating point
specifications were successfully met.
2 states and 1 inputs
```

Double click `op_trim1` to view the resulting operating point. This action opens the Edit trim point dialog box. Select the **Input** tab.



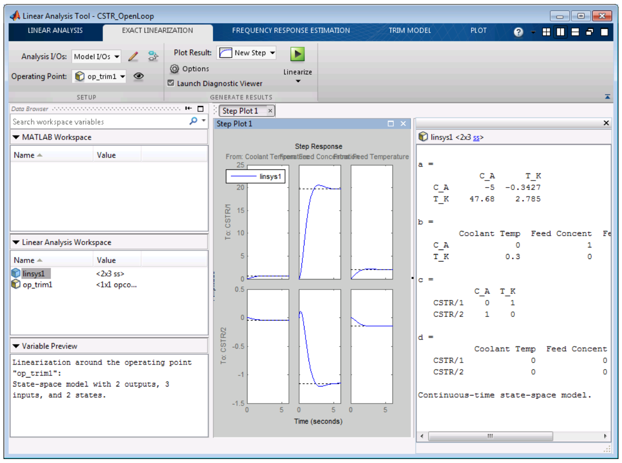
Input	Desired Value	Actual Value
CSTR_OpenLoop/Coolant Temperature		
Input - 1	[-Inf , Inf]	299.0349

The coolant temperature at steady state is 299 K, as desired.

8 In the **Exact Linearization** tab, select `op_trim1` from the **Operating Point** list.

9 Click **Linearize**.

This action creates the linear model **linsys1** in the **Linear Anaysis Workspace**. **linsys1** uses **op_trim1** as its operating point.



```
Linearization around the operating point
"op_trim1":
State-space model with 2 outputs, 3
inputs, and 2 states.
```

```
a =
             C_A      T_K
   C_A        -5  -0.3427
   T_K     47.68    2.785

b =
          Coolant Temp  Feed Concent  Fe
   C_A               0             1
   T_K             0.3             0

c =
            C_A  T_K
   CSTR/1     0    1
   CSTR/2     1    0

d =
            Coolant Temp  Feed Concent
   CSTR/1              0             0
   CSTR/2              0             0

Continuous-time state-space model.
```

The step response from feed concentration to output CSTR/2 displays an interesting inverse response. An examination of the linear model shows that CSTR/2 is the residual CSTR concentration, `C_A`. When the feed concentration increases, `C_A` increases initially (more reactant is entering), which increases the reaction rate. This increases the reactor temperature (output `CSTR/1`), which further increases the reaction rate and `C_A` decreases dramatically.

10 Export the linearization result to the MATLAB workspace.

If necessary, you can repeat any of the above steps. Once you are satisfied with your linear model, drag and drop it from the **Linear Anaysis Workspace** to the **MATLAB Workspace** in the Linear Analysis Tool. You may now use the linearization result with other tools, such as the controller design.

Identify Plant from Data

This example shows how to identify a linear plant model using measured data.

When you have measured plant input/output data, you can use System Identification Toolbox software to estimate a linear plant model. Then, you can use the estimated plant model to create a model predictive controller.

You can estimate the plant model either programmatically or by using the System Identification app.

This example requires a System Identification Toolbox license.

Load the measured data.

```
load dryer2
```

The variables `u2` and `y2`, which contain the data measured for a temperature-control application, are loaded into the MATLAB workspace. `u2` is the plant input, and `y2` is the plant output. The sampling period for `u2` and `y2` is 0.08 seconds.

Create an iddata object for the measured data.

Use an `iddata` object to store the measured values of the plant inputs and outputs, sampling time, channel names, etc.

```
Ts = 0.08;
dry_data = iddata(y2,u2,Ts);
```

`dry_data` is an `iddata` object that contains the measured input/output data.

You can optionally assign channel names and units for the input/output signals. To do so, use the `InputName`, `OutputName`, `InputUnit` and `OutputUnit` properties of an `iddata` object. For example:

```
dry_data.InputName = 'Power';
dry_data.OutputName = 'Temperature';
```

Detrend the measured data.

Before estimating the plant model, preprocess the measured data to increase the accuracy of the estimated model. For this example, `u2` and `y2` contain constant offsets that you eliminate.

```
dry_data_detrended = detrend(dry_data);
```

Estimate a linear plant model.

You can use System Identification Toolbox software to estimate a linear plant model in one of the following forms:

- State-space model
- Transfer function model
- Polynomial model
- Process model
- Grey-box model

For this example, estimate a third-order, linear state-space plant model using the detrended data.

```
plant = ssest(dry_data_detrended,3);
```

To view the estimated parameters and estimation details, at the MATLAB command prompt, type `plant`.

See Also

`detrend` | `iddata` | `ssest`

Related Examples

- “Identify Linear Models Using System Identification App”
- “Design Controller for Identified Plant” on page 2-29
- “Design Controller Using Noise Model of Identified Model” on page 2-31

More About

- “Handling Offsets and Trends in Data”
- “Working with Impulse-Response Models” on page 2-33

Design Controller for Identified Plant

This example shows how to design a model predictive controller for a linear identified plant model.

This example uses only the measured input and output of the identified model for the model predictive controller plant model.

This example requires a System Identification Toolbox license.

Obtain an identified linear plant model.

```
load dryer2;
Ts = 0.08;
dry_data = iddata(y2,u2,Ts);
dry_data_detrended = detrend(dry_data);
plant_idss = ssest(dry_data_detrended,3);
```

`plant_idss` is a third-order, identified state-space model that contains one measured input and one unmeasured (noise) input.

Convert the identified plant model to a numeric LTI model.

You can convert the identified model to an `ss`, `tf`, or `zpk` model. For this example, convert the identified state-space model to a numeric state-space model.

```
plant_ss = ss(plant_idss);
```

`plant_ss` contains the measured input and output of `plant_idss`. The software discards the noise input of `plant_idss` when it creates `plant_ss`.

Design a model predictive controller for the numeric plant model.

```
controller = mpc(plant_ss,Ts);
```

`controller` is an `mpc` object. The software treats:

- The measured input of `plant_ss` as the manipulated variable of `controller`
- The output of `plant_ss` as the measured output of the plant for `controller`

To view the structure of the model predictive controller, type `controller` at the MATLAB command prompt.

See Also

`mpc`

Related Examples

- "Identify Plant from Data" on page 2-27
- "Design Controller Using Noise Model of Identified Model" on page 2-31

Design Controller Using Noise Model of Identified Model

This example shows how to design a model predictive controller that includes the noise model of an identified plant model.

This example requires a System Identification Toolbox license.

Obtain an identified linear plant model.

```
load dryer2;
Ts = 0.08;
dry_data = iddata(y2,u2,Ts);
dry_data_detrended = detrend(dry_data);
plant_idss = ssest(dry_data_detrended,3);
```

`plant_idss` is a third-order, identified state-space model that contains one measured input and one unmeasured (noise) input.

Design a model predictive controller for the identified plant model.

```
controller = mpc(plant_idss,Ts);
```

`controller` is an `mpc` object. The software treats:

- The measured input of `plant_ss` as the manipulated variable of `controller`
- The unmeasured noise input of `plant_ss` as the unmeasured disturbance of the plant for `controller`
- The output of `plant_ss` as the measured output of the plant for `controller`

To view the structure of the model predictive controller, at the MATLAB command prompt, type `controller`.

You can change the treatment of a plant input in one of two ways:

- Programmatic — Use the `setmpcsignals` command to modify the signal .
- Model Predictive Control Toolbox design tool — Use the **Input signal properties** table to modify the plant model signal types.

You can also design a model predictive controller using:

```
plant_ss = ss(plant_idss,'augmented');
controller = mpc(plant_ss,Ts);
```

When you use the `'augmented'` input argument, `ss` creates two input groups, `Measured` and `Noise`, for the measured and noise inputs of `plant_idss`. `mpc` handles the measured and noise components of `plant_ss` and `plant_idss` identically.

See Also

`mpc`

Related Examples

- “Identify Plant from Data” on page 2-27
- “Design Controller for Identified Plant” on page 2-29

Working with Impulse-Response Models

You can use System Identification Toolbox software to estimate finite step-response or finite impulse-response (FIR) plant models using measured data. Such models, also known as *nonparametric models* (see [6] for example), are easy to determine from plant data ([3] and [7]) and have intuitive appeal.

You use the `impulseest` function to estimate an FIR model from measured data. The function returns an identified transfer function model, `idtf`. To design a model predictive controller for the plant, you can convert the identified FIR plant model to a numeric LTI model. However, this conversion usually yields a high-order plant, which can degrade the controller design. This result is particularly an issue for MIMO systems. For example, the estimator design can be affected by numerical precision issues with high-order plants.

Model predictive controllers work best with low-order parametric models (see [5] for example). Therefore, to design a model predictive controller using measured plant data, you can use a parametric estimator, such as `ssest`. Then estimate a low-order parametric plant model. Alternatively, you can initially identify a nonparametric model using the data and then estimate a low-order parametric model for the nonparametric model's response. (See [10] for an example.)

See Also
`impulseest` | `ssest`

Related Examples

- "Identify Plant from Data" on page 2-27
- "Design Controller for Identified Plant" on page 2-29
- "Design Controller Using Noise Model of Identified Model" on page 2-31

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[11] Wood, R. K., and M. W. Berry, *Chem. Eng. Sci.*, Vol. 28, pp. 1707, 1973.

Designing Controllers Using the Design Tool GUI

Design Controller Using the Design Tool

In this section...

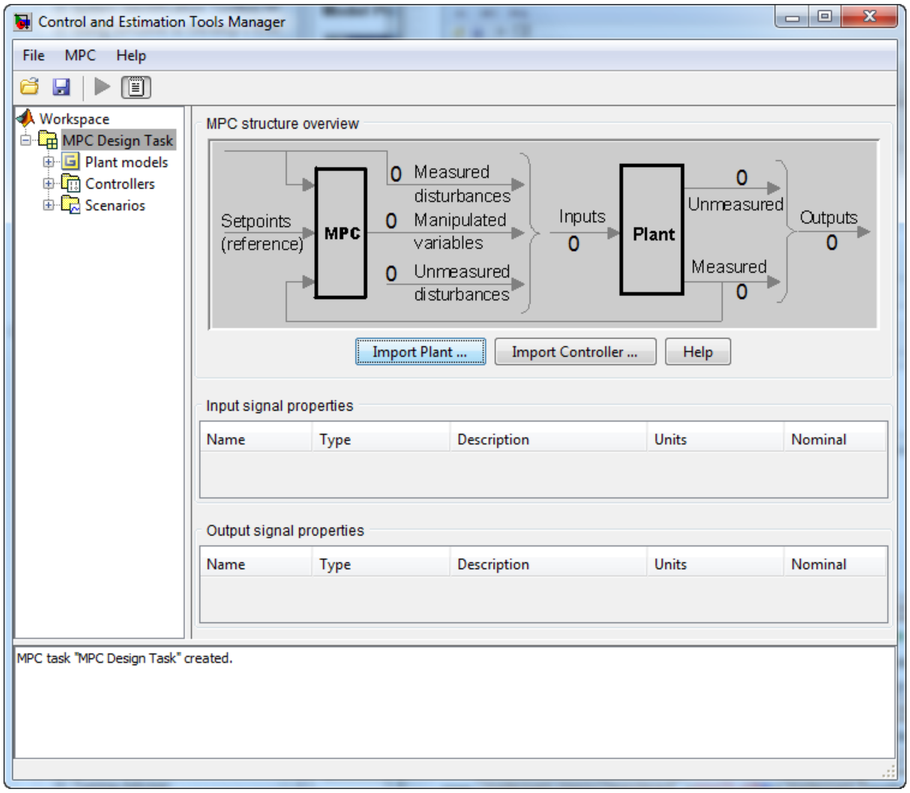
The Model Predictive Control Toolbox design tool is a graphical user interface for controller design. This GUI is part of the Control and Estimation Tools Manager GUI.

Start the Design Tool

Start the design tool by typing the MATLAB command

```
mpctool
```

The Control and Estimation Tools Manager window appears, as shown below. By default, it contains a Model Predictive Control Toolbox task called **MPC Design Task** (listed in the tree view on the left side of the window), which is selected, causing the view shown on the right to appear.



MPC task "MPC Design Task" created.

Model Predictive Control Toolbox Design Tool Initial View

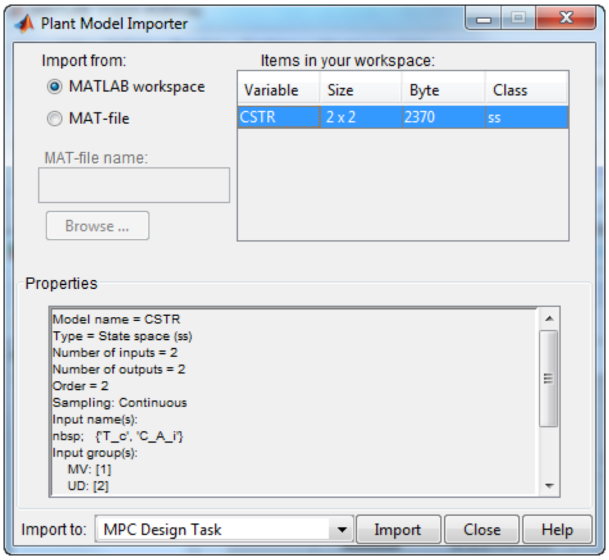
Load a Plant Model

The first step in the design is to load a plant model. Its dimensions and signal characteristics set the context for the remaining steps. You can either load the model directly, as described in this section, or indirectly by importing a controller or a saved design (see “Load Your Saved Work” on page 3-45).

The following example uses the `CSTR` model described in “CSTR Model” on page 2-18. Verify that the LTI object `CSTR` is in your MATLAB workspace.

Plant Model Importer Dialog Box

Click the **Import Plant** button in the design tool's initial view (see Model Predictive Control Toolbox Design Tool Initial View). The Plant Model Importer dialog box appears.



Variable	Size	Byte	Class
CSTR	2 x 2	2370	ss

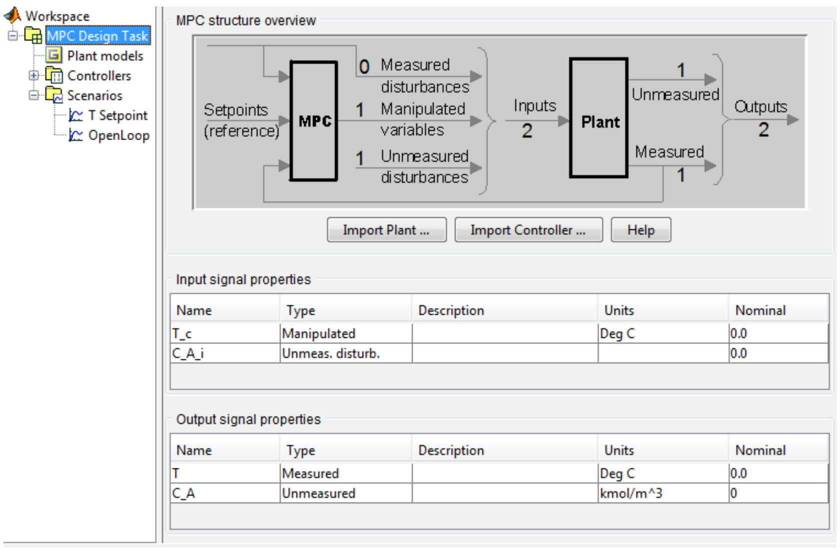
```
Model name = CSTR
Type = State space (ss)
Number of inputs = 2
Number of outputs = 2
Order = 2
Sampling: Continuous
Input name(s):
nbsp;   {'T_c', 'C_A_i'}
Input group(s):
   MV: [1]
   UD: [2]
```

Plant Model Importer Dialog Box

The **Import from MATLAB workspace** option button should be selected by default, as shown. The **Items in your workspace** table lists your LTI models. If `CSTR` doesn't appear, define it as discussed in "State-Space Models" on page 2-10, then reopen this dialog box.

After `CSTR` appears, select it. The **Properties** list displays the number of inputs and outputs, their names and signal types, etc.

Click the **Import** button. This loads `CSTR` into the design tool. Then click the **Close** button (otherwise the dialog box remains visible in case you want to import another model). The design tool should appear as in Model Predictive Control Toolbox Design Tool's Signal Definition View.



Input signal properties

Name	Type	Description	Units	Nominal
T_c	Manipulated		Deg C	0.0
C_A_i	Unmeas. disturb.			0.0

Output signal properties

Name	Type	Description	Units	Nominal
T	Measured		Deg C	0.0
C_A	Unmeasured		kmol/m^3	0

Model Predictive Control Toolbox Design Tool's Signal Definition View

Signal Property Specifications

The figure's graphical display indicates that you've imported a plant model by showing the number of inputs and outputs, and the number in each subclass: measured disturbance, manipulated variables, etc.). Also, the tables labeled **Input signal properties** and **Output signal properties** fill with data:

- The **Name** entries are from the `CSTR` model's `InputName` and `OutputName` properties (the design tool assigns defaults if necessary). You can edit these at any time.
- The **Type** entries are from the `CSTR` model's `InputGroup` and `OutputGroup` properties. (The design tool defaults all unspecified inputs to manipulated variables and all unspecified outputs to measured.)

Note Once you leave this view, if you subsequently change a signal type, you will have to restart the design. Be sure the signal types are correct at the beginning.

- The **Description** and **Unit** entries are optional. You can enter the values shown in Model Predictive Control Toolbox Design Tool's Signal Definition View manually. As you will see, the design tool uses them to label plots and other tables.

- The **Nominal** entries are initial conditions for simulations. The design tool default is `0.0`.

Navigate Using the Tree View

The *tree* in the left-hand frame of Model Predictive Control Toolbox Design Tool's Signal Definition View shows that the default **MPC Design Task** *node* (see Model Predictive Control Toolbox Design Tool Initial View) has been renamed **CSTRcontrol** by clicking the name, waiting for the usual edit box to appear, typing the new name, and pressing **Enter** to finalize the choice.

Model Predictive Control Toolbox Design Tool's Signal Definition View also shows three new nodes below **CSTRcontrol**. These activate once you've imported a plant model (or controller).

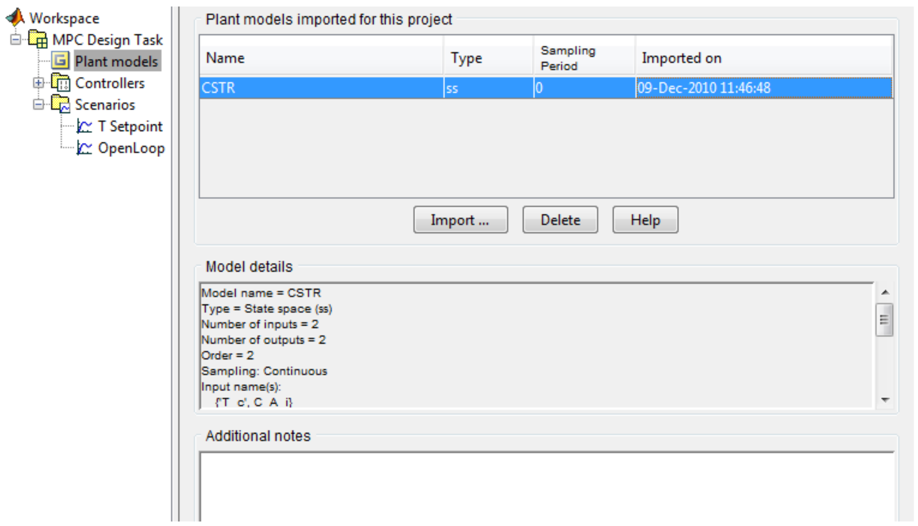
In general, clicking a node displays a *view* supporting a particular design activity.

Project View (Signal Properties Tables)

For example, clicking the *project* node (**CSTRcontrol** in Model Predictive Control Toolbox Design Tool's Signal Definition View) allows you to review and edit the signal properties tables.

Listing Your Plant Models

Select the **Plant models** node to list the plant models you've imported, as shown in Plant Models View with CSTR Model Selected. (Each model name is editable.) The **Model details** section displays properties of the selected model. There is also a space to enter notes describing the model's special features. Buttons allow you to import a new model or delete one you no longer need.



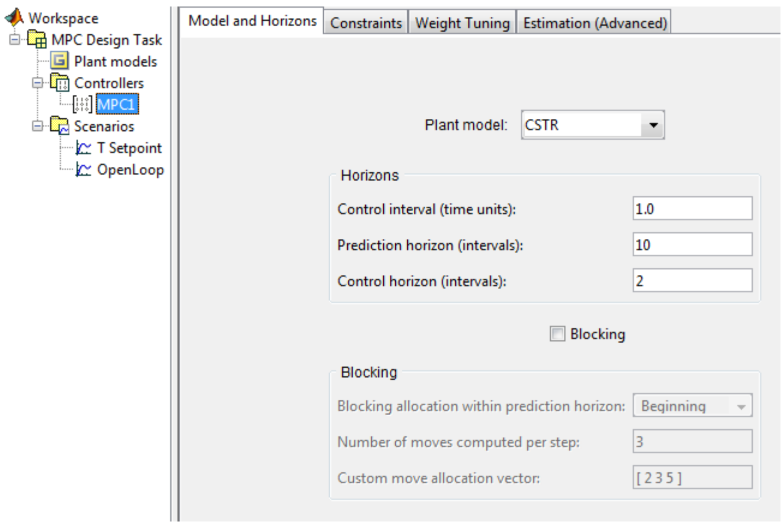
Name	Type	Sampling Period	Imported on
CSTR	ss	0	09-Dec-2010 11:46:48

```
Model name = CSTR
Type = State space (ss)
Number of inputs = 2
Number of outputs = 2
Order = 2
Sampling: Continuous
Input name(s):
   {'T c', C A i}
```

Plant Models View with CSTR Model Selected

Viewing Your Controllers

Next, select **Controllers**. The view shown in Controllers View appears. A **+** sign to the left of **Controllers** indicates that it contains subnodes. You can click a **+** sign to expand the tree (as shown in Controllers View, where the **+** sign has changed to a – sign).



Plant model: CSTR

Horizons

Control interval (time units): 1.0

Prediction horizon (intervals): 10

Control horizon (intervals): 2

Blocking

Blocking

Blocking allocation within prediction horizon: Beginning

Number of moves computed per step: 3

Custom move allocation vector: [2 3 5]

Controllers View

The table at the top of Controllers View lists all the controllers you've defined. The design tool automatically creates a controller containing Model Predictive Control Toolbox defaults, naming it **MPC1**. It is a subnode of **Controllers**.

Note If you define additional controllers, they will appear here. For example, you might want to test several options, saving each as a separate controller, making it easy to switch from one to another during testing.

The table **Controllers defined in this project** allows you to edit the controller name and gives you quick access to three important design parameters: **Plant Model**, **Control Interval**, and **Prediction Horizon**. All are editable, but leave them at their default values for this example.

The buttons shown in Controllers View let you do the following:

- Import a controller designed previously and stored either in your workspace or in a MAT-file.
- Export the selected controller to your workspace.
- Create a new controller initialized to Model Predictive Control Toolbox defaults.

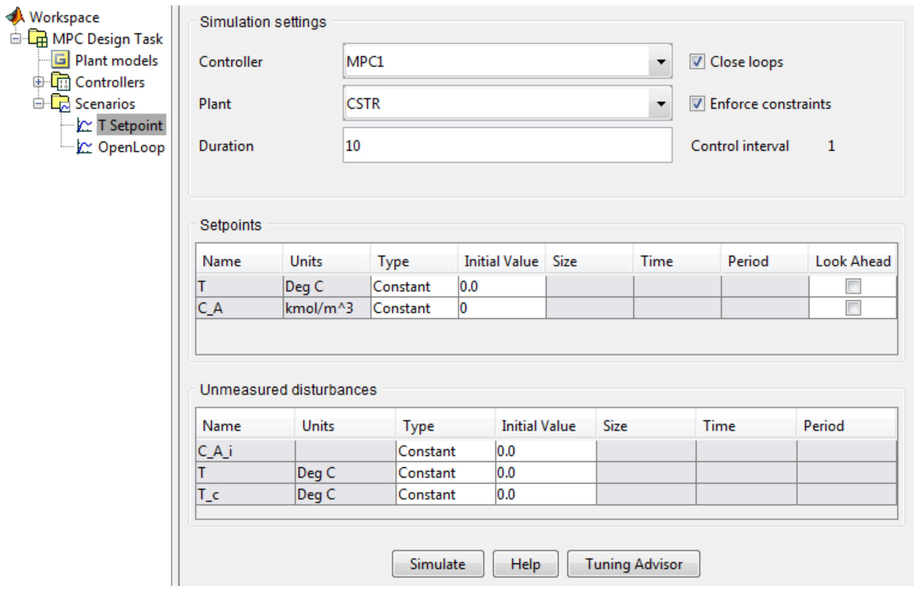
- Copy the selected controller, creating a duplicate you can modify.
- Delete the selected controller.

You can also right-click the **Controllers** node to access menu options **New**, **Import**, and **Export**, or one of its subnodes to access menu options **Copy**, **Rename**, **Export**, and **Delete**.

Select the **MPC1** node to display Model Predictive Control Toolbox default controller settings. ("Change Controller Settings" on page 3-22 covers this view in detail).

Viewing Simulation Scenarios

A *scenario* is a set of conditions defining a simulation. The design tool creates a default scenario and names it **Scenario1**. To view it, click the **+** symbol next to the **Scenarios** node, and select the **Scenario1** subnode. You should see a view like that shown in CSTR Temperature Setpoint Change Scenario.



Simulation settings

Controller	MPC1	✓ Close loops	
Plant	CSTR	✓ Enforce constraints	
Duration	10	Control interval	1

Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
T	Deg C	Constant	0.0				☐
C_A	kmol/m^3	Constant	0				☐

Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i		Constant	0.0			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

Scenarios View

Whenever you select the **Scenarios** node, you see a table summarizing your current scenarios (not shown). Its function is similar to the **Controllers** view described previously.

Perform Linear Simulations

You usually want to test your controller in simulations. The Model Predictive Control Toolbox design tool makes it easy to run closed-loop simulations involving a Model Predictive Control Toolbox controller and an LTI plant model. This plant can differ from that used in the controller design, allowing you to test your controller's sensitivity to prediction errors (see “Test Controller Robustness” on page 3-47).

This section covers the following topics:

- “Defining Simulation Conditions” on page 3-10
- “Running a Simulation” on page 3-11
- “Open-Loop Simulations” on page 3-14

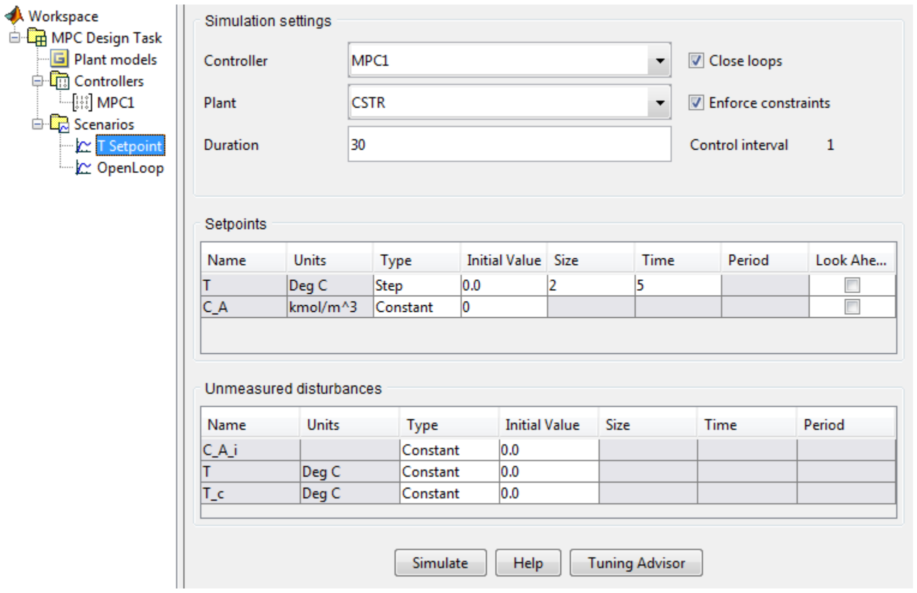
Defining Simulation Conditions

To define simulation conditions, select an existing scenario node or create a new one, and then edit its tabular fields to define your conditions.

CSTR Temperature Setpoint Change Scenario shows the result of renaming the default **Scenario1** node to **T Setpoint** and editing its default conditions. The required editing steps are as follows:

- Increase **Duration** from `10` to `30`.
- Locate the tabular data defining the reactor temperature setpoint (first row of the upper table in CSTR Temperature Setpoint Change Scenario).
  - Click the **Type** table cell and select `Step` from the list of choices.
  - Change **Size** from `1.0` to `2`.
  - Change **Time** from `1.0` to `5`.

Note The **Control Interval** is a property of the controller being used (`MPC1` in this case). To change it, select the controller node in the tree, and then edit the value on the **Model and Horizons** tab (see “Model and Horizons” on page 3-22). Such a change would apply to all simulations involving that controller.



Simulation settings

Controller	MPC1	✓ Close loops	
Plant	CSTR	✓ Enforce constraints	
Duration	30	Control interval	1

Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahe...
T	Deg C	Step	0.0	2	5		☐
C_A	kmol/m^3	Constant	0				☐

Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i		Constant	0.0			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

CSTR Temperature Setpoint Change Scenario

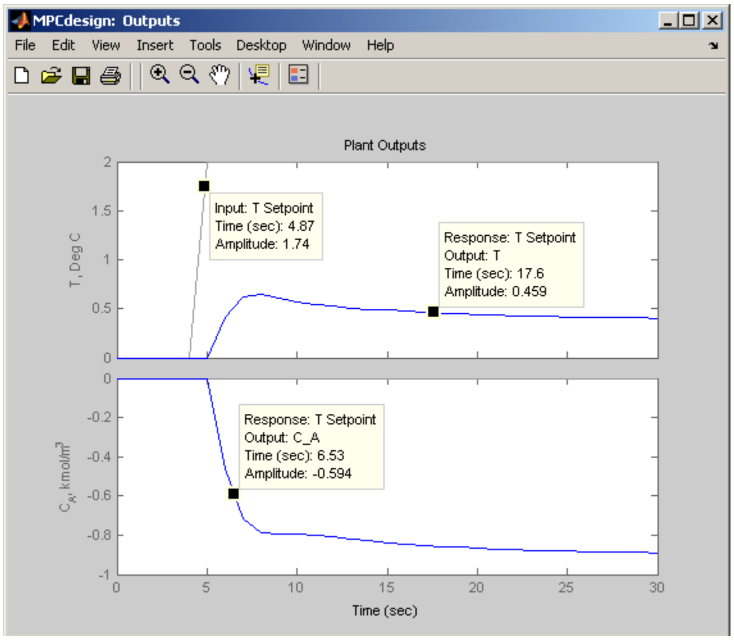
Running a Simulation

To run a simulation, do *one* of the following:

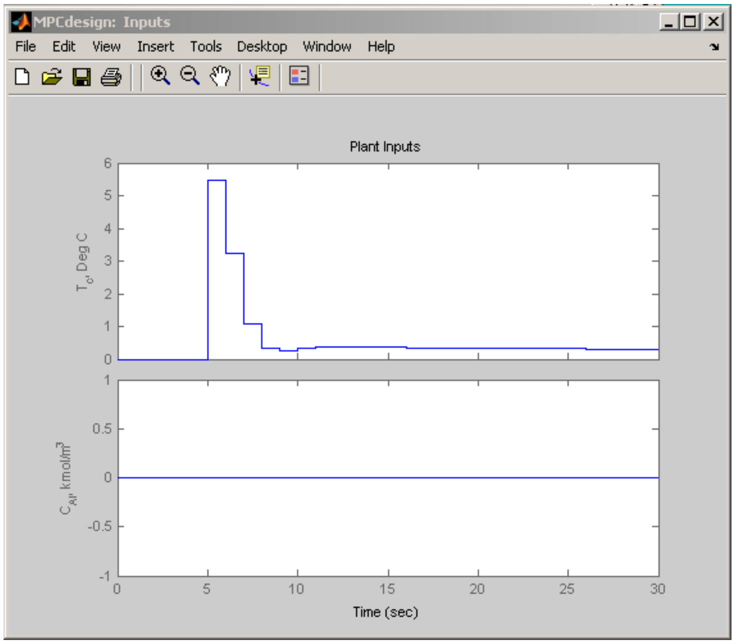
- Select the scenario you want to run, and click its **Simulate** button (see the bottom of CSTR Temperature Setpoint Change Scenario).
- Click the toolbar's **Simulation** button, which is the triangular icon shown on the top left of CSTR Temperature Setpoint Change Scenario.

The toolbar method runs the *current scenario*, i.e., the one most recently selected or modified.

Try running the **T Setpoint** scenario. This should generate the two response plot windows shown in Plant Outputs for T Setpoint Scenario with Added Data Markers and Plant Inputs for the T Setpoint Scenario.



Plant Outputs for T Setpoint Scenario with Added Data Markers



Plant Inputs for the T Setpoint Scenario

Plant Outputs for T Setpoint Scenario with Added Data Markers shows that the reactor temperature setpoint increases suddenly by 2 degrees at $t = 5$, as you specified when defining the scenario in "Defining Simulation Conditions" on page 3-10. Unfortunately, the temperature does not track the setpoint very well, and there is a persistent error of about 1.6 degrees at the end of the simulation.

Also, the controller requests a sudden jump in the coolant temperature (see the upper graph in Plant Inputs for the T Setpoint Scenario), which might be difficult to deliver in practice. (The lower graph in Plant Inputs for the T Setpoint Scenario shows that the feed concentration, C_{Ai}, remains constant, as specified in the scenario.)

See "Change Controller Settings" on page 3-22 for ways to overcome these deficiencies.

Note Plant Outputs for T Setpoint Scenario with Added Data Markers has *data markers*. To add these, left-click the curve to create the data marker. Drag a marker to relocate it. Left-click in a graph's white space to erase its markers. For more information on data markers, see the Control System Toolbox documentation.

Open-Loop Simulations

By default, scenarios are *closed loop*, i.e., an active controller adjusts the manipulated variables entering your plant based on feedback from the plant outputs. You can also run *open-loop* simulations that test the plant model without feedback control.

For example, you might want to check your plant model's response to a particular input without opening another tool. You might also want to display unmeasured disturbance signals before using them in a closed-loop simulation.

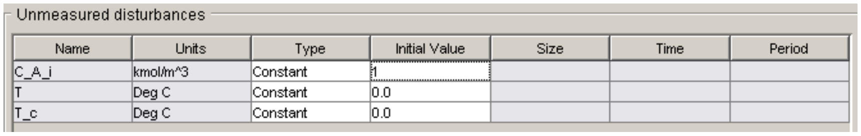
To see how this works, create a new scenario by right-clicking the **T Setpoint** node in the tree, and selecting **Copy Scenario** in the resulting menu. Rename the copy **OpenLoop**.

Select **OpenLoop** in the tree. On its scenario view, change Duration to `100`, and turn off (clear) **Close loops**.

Open-loop simulations ignore the **Setpoints** table settings, so there's no need to modify them.

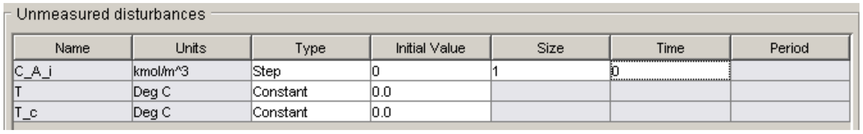
If `CSTR` had a measured disturbance input, the pane would contain another table allowing you to specify it.

For this example, focus on the **Unmeasured disturbances** table. Configure it as shown below.


Unmeasured disturbances

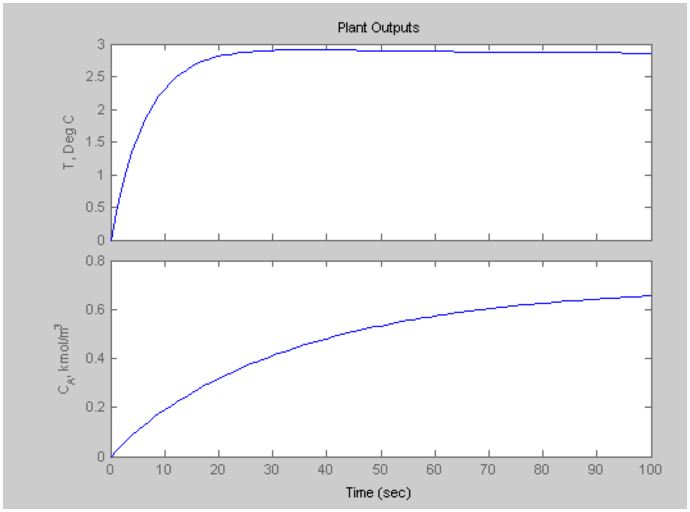
Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Constant	1			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

The `C_A_i` input's nominal value is `0.0` (see Model Predictive Control Toolbox Design Tool's Signal Definition View), so the above models a sudden increase to `1` at the beginning of the simulation. The following is an equivalent setup using the `Step` type.


Unmeasured disturbances

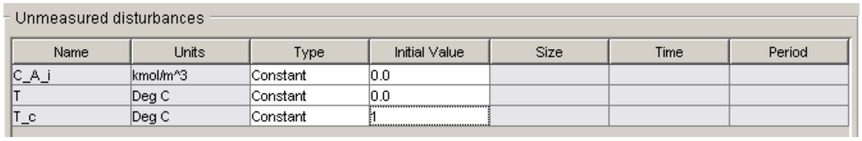
Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Step	0	1	0	
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

Using one of these, simulate the scenario (click its **Simulate** button). The output response plot should be as shown below.



This is the `CSTR` model's open-loop response to a unit step in the C_{Ai} disturbance input.

You could also set up the table as shown below.

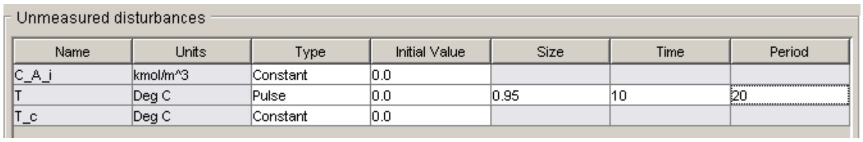


Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Constant	0.0			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	1			

This simulation would display the open-loop response to a unit step in the T_c manipulated variable input (try it).

Finally, set it up as follows.

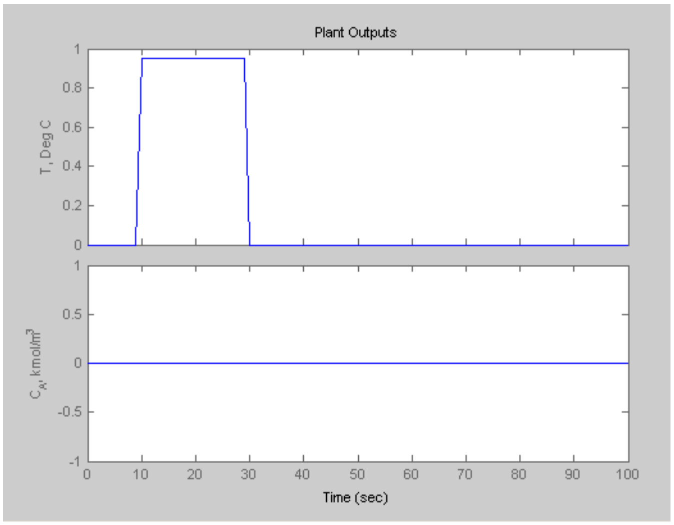


Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Constant	0.0			
T	Deg C	Pulse	0.0	0.95	10	20
T_c	Deg C	Constant	0.0			

This adds a pulse to the T output. The pulse begins at time $t = 10$, and lasts 20 time units. Its height is 0.95 degrees.

Run the simulation. The output response plot displays the pulse (see Response Plot Showing Open-Loop Pulse Disturbance). In this case, the T output's nominal value is zero, so you only see the pulse. (If the T output had a nonzero nominal value, the pulse would add to that.)

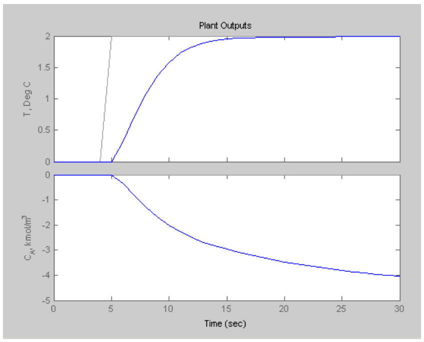


Response Plot Showing Open-Loop Pulse Disturbance

If you were to run a *closed-loop* simulation with this same T disturbance, the controller would attempt to hold T at its setpoint, and the result would differ from that shown in Response Plot Showing Open-Loop Pulse Disturbance.

Customize Response Plots

Each time you simulate a scenario, the design tool plots the corresponding plant input and output responses. The graphic below shows such a *response plot* for a plant having two outputs (the corresponding input response plot is not shown).



By default, each plant signal plots in its own graph area (as shown above). If the simulation is closed loop, each output signal plot include the corresponding setpoint.

The following sections describe response plot customization options:

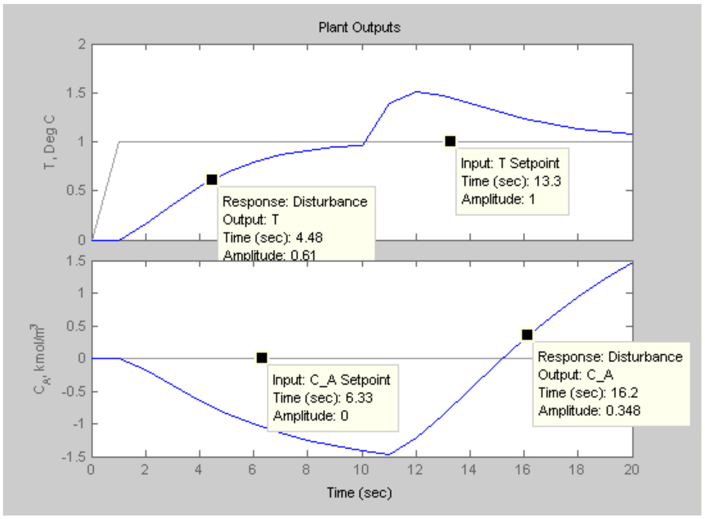
- "Data Markers" on page 3-17
- "Displaying Multiple Scenarios" on page 3-19
- "Viewing Selected Variables" on page 3-20
- "Grouping Variables in a Single Plot" on page 3-20
- "Normalizing Response Amplitudes" on page 3-21

Data Markers

You can use data markers to label a curve or to display numerical details.

Adding a Data Marker

To add a data marker, click the desired curve at the location you want to mark. The following graph shows a marker added to each output response and its corresponding setpoint.



Data Marker Contents

Each data marker provides information about the selected point, as follows:

- **Response** – The *scenario* that generated the curve.
- **Time** – The time value at the data marker location.
- **Amplitude** – The signal value at the data marker location.
- **Output** – The plant variable name (plant outputs only).
- **Input** – Variable name for plant inputs and setpoints.

Changing a Data Marker's Alignment

To relocate the data marker's label (without moving the marker), right-click the marker, and select one of the four **Alignment** menu options. The above example shows three of the possible four alignment options.

Relocating a Data Marker

To move a marker, left-click it (holding down the mouse key) and drag it along its curve to the desired location.

Deleting Data Markers

To delete all data markers in a plot, click in the plot's white space.

To delete a single data marker, right-click it and select the **Delete** option.

Right-Click Options

Right-click a data marker to use one of the following options:

- **Alignment** – Relocate the marker's label.
- **Font Size** – Change the label's font size.
- **Movable** – On/off option that makes the marker movable or fixed.
- **Delete** – Deletes the selected marker.
- **Interpolation** – Interplolate linearly between the curve's data points, or locate at the nearest data point.
- **Track Mode** – Changes the way the marker responds when you drag it.

Displaying Multiple Scenarios

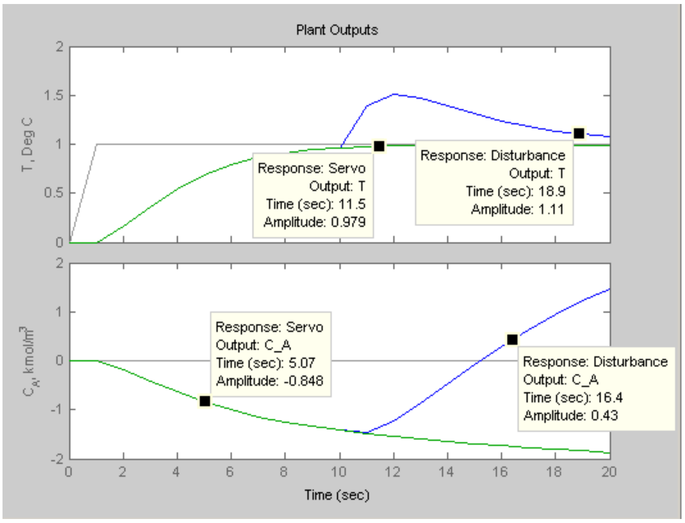
By default the response plots include all the scenarios you've simulated. The example below shows a response plot for a plant with two outputs. The data markers indicate the two scenarios being plotted: “Accurate Model” and “Perturbed Model”. Both scenarios use the same setpoints (not marked—the lighter solid lines).

Viewing Selected Scenarios

If your plots are too cluttered, you can hide selected scenarios. To do so:

- Right-click in the plot's white space.
- Select **Responses** from the resulting context menu.
- Toggle a response on or off using the submenu.

Note This selection affects all variables being plotted.



Revising a Scenario

If you modify and recalculate a scenario, its data are replotted, replacing the original curves.

Viewing Selected Variables

By default, the design tool plots all plant inputs in a single window, and plots all plant outputs in another. If your application involves many signals, the plots of each may be too small to view comfortably.

Therefore, you can control the variables being plotted. To do so, right-click in a plot's white space and select **Channel Selector** from the resulting menu. A dialog box appears, on which you can opt to show or hide each variable.

Grouping Variables in a Single Plot

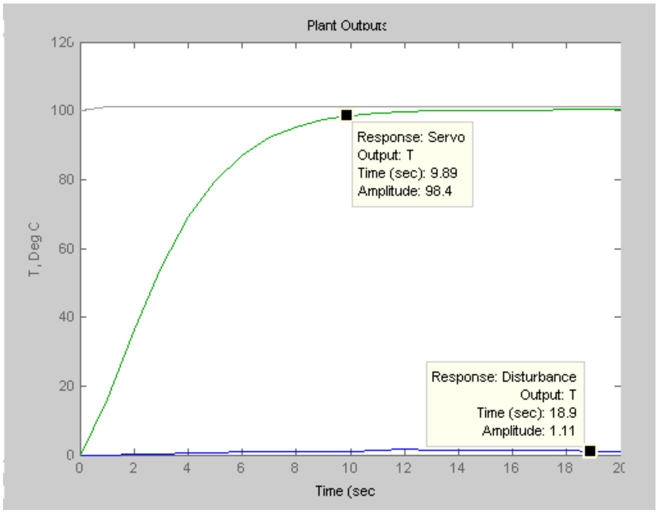
By default, each variable appears in its own plot area. You can instead choose to display variables together in a single plot. To do so, right-click in a plot's white space, and select **Channel Grouping**, and then select **All**.

To return to the default mode, use the **Channel Grouping**: **None** option.

Normalizing Response Amplitudes

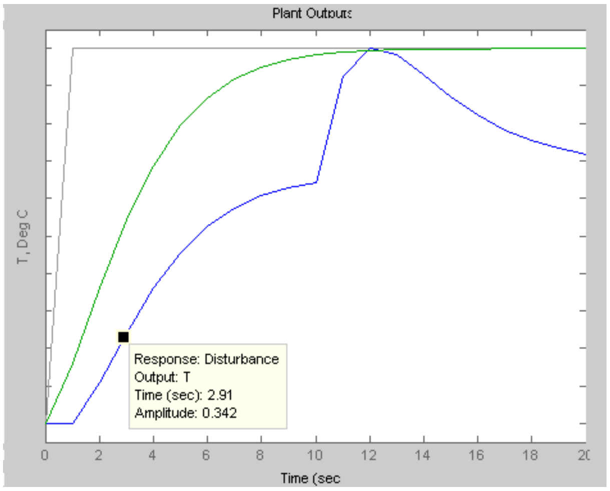
When you're using the **Channel Grouping**: **All** option, you might find that the variables have very different scales, making it difficult to view them together. You can choose to *normalize* the curves, so that each expands or contracts to fill the available plot area.

For example, the plot below shows two plant outputs together (**Channel Grouping**: **All** option). The outputs have very different magnitudes. When plotted together, it's hard to see much detail in the smaller response.



The plot below shows the normalized version, which displays each curve's variations clearly.

The y-axis scale is no longer meaningful, however. If you want to know a normalized signal's amplitude, use a data marker (see “Adding a Data Marker” on page 3-17). Note that the two data markers on the plot below are at the same normalized y-axis location, but correspond to very different amplitudes in the original (unnormalized) coordinates.



Change Controller Settings

The simulations shown in Plant Outputs for T Setpoint Scenario with Added Data Markers and Plant Inputs for the T Setpoint Scenario uses the default controller settings. These often work well, but the `CSTR` application is an exception. The following discussion covers the main controller options in detail, and shows how to tune a controller for better performance.

This section covers the following topics:

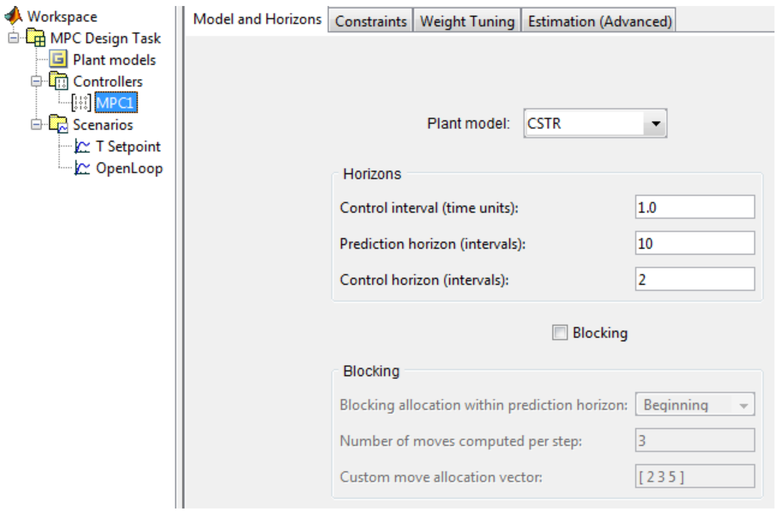
- "Model and Horizons" on page 3-22
- "Weight Tuning" on page 3-23
- "Blocking" on page 3-28
- "Defining Manipulated Variable Constraints" on page 3-31
- "Disturbance Modeling and Estimation" on page 3-33

Model and Horizons

Select your **MPC1** controller in the tree. The view shown in Controller Options — Model and Horizons Tab should appear. If necessary, click the **Model and Horizons** tab to bring it to the front. This tab contains the following controller options:

- **Plant model** specifies the LTI model to be used for controller predictions.
- **Control interval** sets the elapsed time between successive adjustments of the controller's manipulated variables.
- **Prediction horizon** is the number of control intervals over which the outputs are to be optimized.
- **Control horizon** sets the number of control intervals over which the manipulated variables are to be optimized.
- Selecting the **Blocking** option gives you more control over the way in which the controller's *moves* are allocated.

Leave all of the **Model and Horizons** tab settings at their default values for now.



Plant model: CSTR

Control interval (time units): 1.0

Prediction horizon (intervals): 10

Control horizon (intervals): 2

☐ Blocking

Blocking allocation within prediction horizon: Beginning

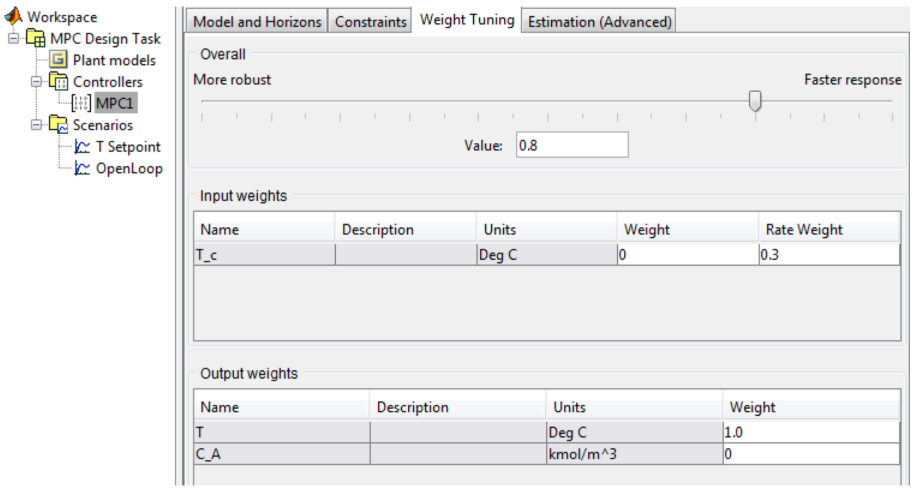
Number of moves computed per step: 3

Custom move allocation vector: [2 3 5]

Controller Options — Model and Horizons Tab

Weight Tuning

Click the **Weight Tuning** tab. The view shown in Controller Options — Weight Tuning Tab appears.



Overall

More robust — Faster response

Value: 0.8

Input weights

Name	Description	Units	Weight	Rate Weight
T_c		Deg C	0	0.3

Output weights

Name	Description	Units	Weight
T		Deg C	1.0
C_A		kmol/m^3	0

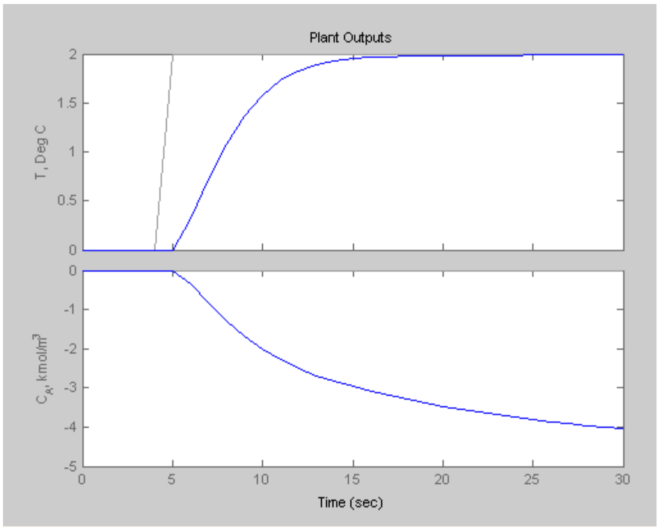
Controller Options — Weight Tuning Tab

Make the following changes (already done in the above view):

- In the **Input weights** section, change the coolant temperature's **Rate Weight** from the default `0.1` to `0.3`.
- In the **Output weights** section, change the reactant concentration's **Weight** (last entry in the second row) from the default `1.0` to `0`.

Test these changes using the **T Setpoint** scenario (click the toolbar's **Simulation** button).

Improved Setpoint Tracking for CSTR Temperature shows that the CSTR temperature now tracks the setpoint change smoothly, reaching the new value in about 10 time units with no overshoot.



Improved Setpoint Tracking for CSTR Temperature

On the other hand, the reactant concentration, C_A, exhibits a larger deviation from its setpoint, which is being held constant at zero (compare to Plant Outputs for T Setpoint Scenario with Added Data Markers, where the final deviation is about a factor of 4 smaller).

This behavior reflects an unavoidable trade-off. The controller has only one adjustment at its disposal: the coolant temperature. Therefore, it can't satisfy setpoints on both outputs.

Output Weights

The output weights let you dictate the accuracy with which each output must track its setpoint. Specifically, the controller predicts deviations for each output over the prediction horizon. It multiplies each deviation by the output's weight value, and then computes the weighted sum of squared deviations, $S_y(k)$, as follows

$$S_y(k) = \sum_{i=1}^{P} \sum_{j=1}^{n_y} \left\{ w^y{}_j [r_j(k+i) - y_j(k+i)] \right\}^2$$

where k is the current sampling interval, $k + i$ is a future sampling interval (within the prediction horizon), P is the number of control intervals in the prediction horizon, n_y is the number of plant outputs, $w^y{}_j$ is the weight for output j, and the term $[r_j(k + i) – y_j(k + i)]$ is a predicted deviation for output j at interval $k + 1$.

The weights must be zero or positive. If a particular weight is large, deviations for that output dominate $S_y(k)$. One of the controller's objectives is to *minimize* $S_y(k)$. Thus, a large weight on a particular output causes the controller to minimize deviations in that output (relative to outputs having smaller weights).

For example, the default values used to produce Plant Outputs for T Setpoint Scenario with Added Data Markers specify equal weights on each output, so the controller is trying to eliminate deviations in both, which is impossible. On the other hand, the design of Improved Setpoint Tracking for CSTR Temperature uses a weight of zero on the second output, so it is able to eliminate deviations in the first output.

Note The second output is unmeasured. Its predictions rely on the plant model and the temperature measurements. If the model were reliable, we could hold the predicted concentration at a setpoint, allowing deviations in the reactor temperature instead. In practice, it would be more common to control the temperature as done here, using the predicted reactant concentration as an auxiliary indicator.

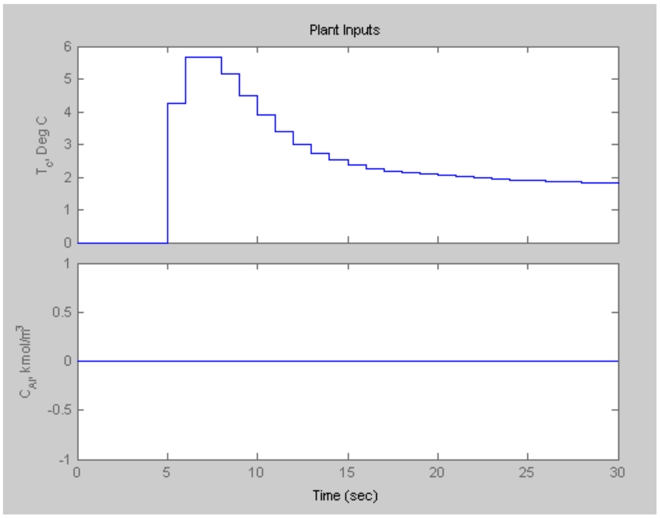
You might expect equal output weights to result in equal output deviations at steady state. Plant Outputs for T Setpoint Scenario with Added Data Markers shows that this is not the case. The reason is that the controller is trying to satisfy several additional objectives simultaneously.

Rate Weights

One is to minimize the weighted sum of controller adjustments, calculated according to

$$S_{\Delta u}(k) = \sum_{i=1}^{M} \sum_{j=1}^{n_{mv}} \left\{ w_j^{\Delta u} \Delta u_j(k+i-1) \right\}^2$$

where M is the number of intervals in the control horizon, n_{mv} is the number of manipulated variables, $\Delta u_j(k+i-1)$ is the predicted adjustment in manipulated variable j at future (or current) sampling interval $k + i – 1$, and $w_j^{\Delta u}$ is the weight on this adjustment, called the *rate weight* because it penalizes the incremental change rather than the cumulative value. Increasing this weight forces the controller to make smaller, more cautious adjustments.



Plant Inputs for Modified Rate Weight

Plant Inputs for Modified Rate Weight shows that increasing the rate weight from 0.1 to 0.3 decreases the move sizes significantly, especially the initial move (compare to Plant Inputs for the T Setpoint Scenario).

Setting the rate weight to `0.1` yields a faster approach to the T setpoint with a small overshoot, but the initial T_c move is about six times larger than needed to achieve the new steady state, which would be unacceptable in most applications (not shown; try it).

Note The controller minimizes the sum $S_y(k) + S_{\Delta u}(k)$. Changes in the coolant temperature have unequal effects on the two outputs, so the steady-state output deviations won't be equal, even when both output weights are unity as in Plant Outputs for T Setpoint Scenario with Added Data Markers.

Input Weights

The controller also minimizes the weighted sum of manipulated variable deviations from their nominal values, computed according to

$$S_u(k) = \sum_{i=1}^{M} \sum_{j=1}^{n_{mv}} \left\{ w_j^u [u_j(k+i-1) - \bar{u}_j] \right\}^2$$

where $w^u{}_j$ is the *input weight* and $\bar{u}_j$ is the nominal value for input *j*. In the above simulations, you used the default, $w^u{}_j = 0$. This is the usual choice.

When a sustained disturbance or setpoint change occurs, the manipulated variable must deviate permanently from its nominal value (as shown in Plant Inputs for the T Setpoint Scenario and Plant Inputs for Modified Rate Weight). Using a nonzero input weight forces the corresponding input back toward its nominal value. Test this by running a simulation in which you set the input weight to 1. The final T_c value is closer to its nominal value, but this causes T to deviate from the new setpoint (not shown).

Note Some applications involve more manipulated variables than plant outputs. In such cases, it is common to define nonzero input weights on certain manipulated variables in order to hold them near their most economical values. The remaining manipulated variables eliminate steady-state error in the plant outputs.

Blocking

The section "Weight Tuning" on page 3-23 used *penalty weights* to shape the controller's response. This section covers the following topics:

- An alternative to penalty weighting, called *blocking*
- Side-by-side controller comparisons

To begin, select **Controllers** in the tree, and click the **New** button, creating a controller initialized to the Model Predictive Control Toolbox default settings. Rename this controller **Blocking 1** by editing the appropriate table cell.

Select **Blocking 1** in the tree, select its **Weight Tuning** tab, and set the **Weight** for output `C_A` to `0` (see "Weight Tuning" on page 3-23 to review the reason for this). Leave other weights at their defaults.

Now select the **Model and Horizons** tab, and select its **Blocking** check box. This activates the blocking options. It also deactivates the **Control Horizon** option (the blocking options override it).

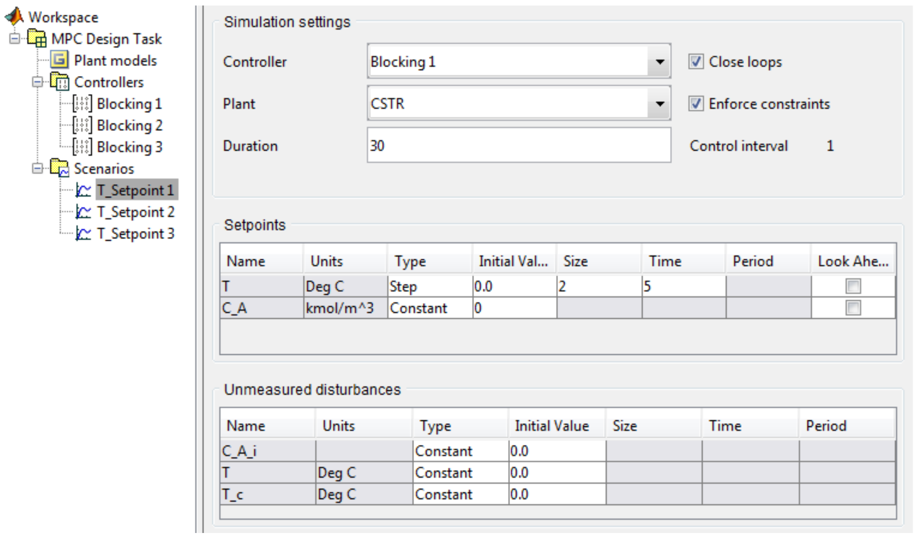
Set **Number of moves computed per step** to 2. Verify that **Blocking allocation within prediction horizon** is set to `Beginning`, the default.

Select **Controllers** in the tree, and use its **Copy** button to create two controllers based on **Blocking 1**. Rename these **Blocking 2** and **Blocking 3**. Edit their blocking options, setting **Blocking allocation within prediction horizon** to `Uniform` for **Blocking 2**, and to `End` for **Blocking 3**.

Select **Scenarios** in the tree. Rename the **T Setpoint** scenario to **T Setpoint 1**, and set its **Controller** option to **Blocking 1**.

Create two copies of **T Setpoint 1**, naming them **T Setpoint 2** and **T Setpoint 3**. Set their **Controller** options to **Blocking 2** and **Blocking 3**, respectively. Now you should have three scenarios, identical except for the controller being used.

Delete the **MPC1** controller and select **T Setpoint 1** in the tree. Your view should resemble T Setpoint 1 Scenario, with three controllers and three scenarios in the tree.



Simulation settings

Controller	Blocking 1	✓ Close loops	
Plant	CSTR	✓ Enforce constraints	
Duration	30	Control interval	1

Setpoints

Name	Units	Type	Initial Val...	Size	Time	Period	Look Ahe...
T	Deg C	Step	0.0	2	5		☐
C_A	kmol/m^3	Constant	0				☐

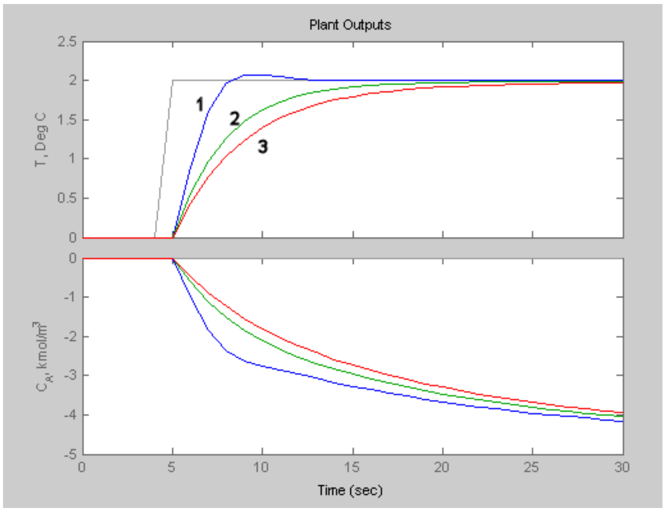
Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i		Constant	0.0			
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

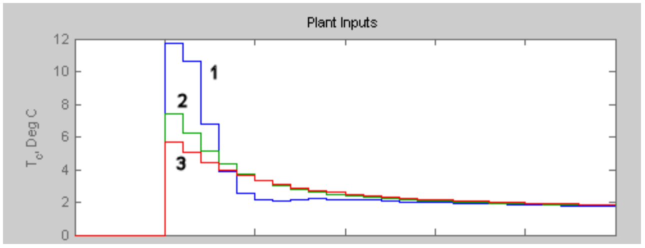
T Setpoint 1 Scenario

If any simulation plot windows are open, close them. This forces subsequent simulations to generate new plots.

Simulate each of the three scenarios. When you run the first, new plot windows open. Leave them open when you run the other two scenarios so all three results appear together, as shown in Blocking Comparison, Outputs and Blocking Comparison, Manipulated Variable.



Blocking Comparison, Outputs



Blocking Comparison, Manipulated Variable

The numeric annotations on these figures refer to the three scenarios. Recall that **T Setpoint 1** uses the default blocking, which usually results in faster setpoint tracking but larger manipulated variable moves. The blocking options in **T Setpoint 2** and **T Setpoint 3** reduce the move size but make setpoint tracking more sluggish.

Results for **T Setpoint 3** are very similar to those shown in Improved Setpoint Tracking for CSTR Temperature and Plant Inputs for Modified Rate Weight, where a penalty *rate weight* reduced the move sizes. If rate weights and blocking achieve the same ends, why does the toolbox provide both features? One difference not evident in this simple problem is that blocking applies to all the manipulated variables in your application, but each rate weight affects one only.

Note To obtain the dashed lines shown for **T Setpoint 2**, activate the plot window and select **Property Editor** from the **View** menu. The Property Editor appears at the bottom of the window. Then select the curve you want to edit. The Property Editor lets you change the line type, thickness, color, and symbol type. Select the axis labels to see additional options.

By default, the toolbox plots each scenario on the same plot. If you recalculate a revised scenario, it replots that result but doesn't change any others.

If you don't want to see a particular scenario, right-click the plot and use the **Responses** menu option to hide it. (You can also close the plot window and recalculate selected responses in a fresh window.)

Defining Manipulated Variable Constraints

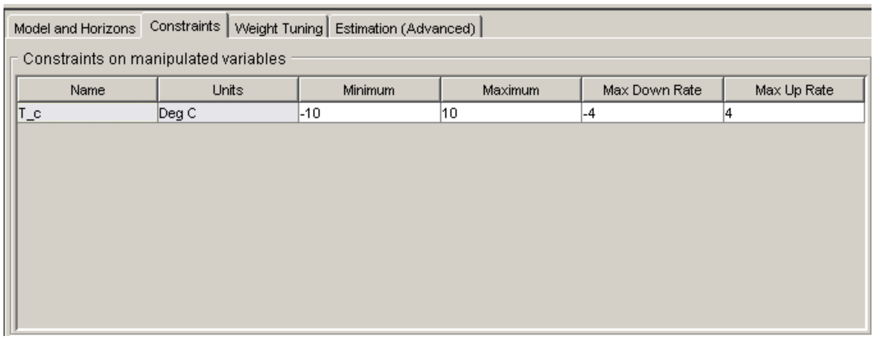
Physical devices have limited ranges and rates of change. For example, the `CSTR` model's coolant might be restricted to a 20 degree range (from –10 to 10) and its maximum rate of change might be ±4 degrees per control interval. If these are true physical restrictions, it's good practice to include them in the controller design. Otherwise the controller might attempt an unrealistic adjustment.

To compare constrained and unconstrained performance for the `CSTR` example, select the **Blocking 1** controller in the tree, and rename it **Unconstrained**.

Select its **Model and Horizons** tab and turn off (clear) its **Blocking** option. Increase **Control horizon** to `3`, and reduce **Control Interval** to `0.25`.

Delete the **Blocking 2** and **Blocking 3** controllers. (Click **Yes** or **OK** to dismiss the resulting warning messages.)

Copy the **Unconstrained** controller. Name this copy **MVconstraints**, and select its **Constraints** tab. Then enter the manipulated variable constraints shown in Entering CSTR Manipulated Variable Constraints.



Constraints on manipulated variables

Name	Units	Minimum	Maximum	Max Down Rate	Max Up Rate
T_c	Deg C	-10	10	-4	4

Entering CSTR Manipulated Variable Constraints

If any simulation plot windows are open, close them (to force fresh plots).

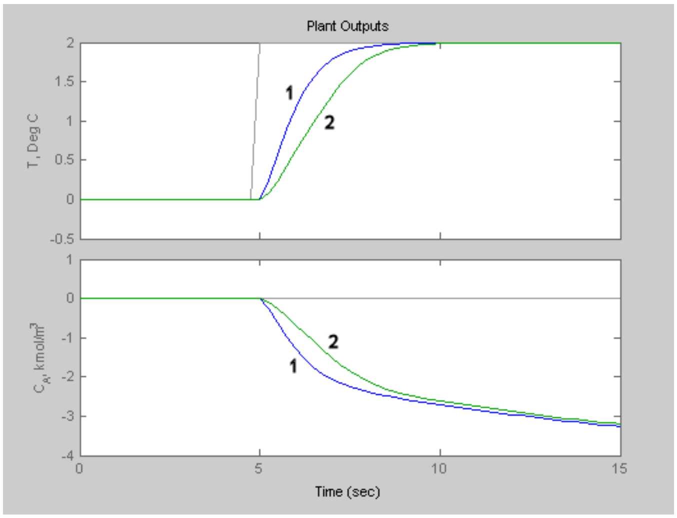
Select the **T Setpoint 1** scenario. If necessary, set its **Controller** option to `Unconstrained`. Change **Duration** to `15`, and simulate the scenario.

Select the **T Setpoint 2** scenario, set its **Controller** option to `MVconstraints`, change its **Duration** to `15`, and simulate it. The results appear in CSTR Outputs, Unconstrained (1) and MVconstraints (2) and CSTR Manipulated Variable, Unconstrained (1) and MVconstraints (2).

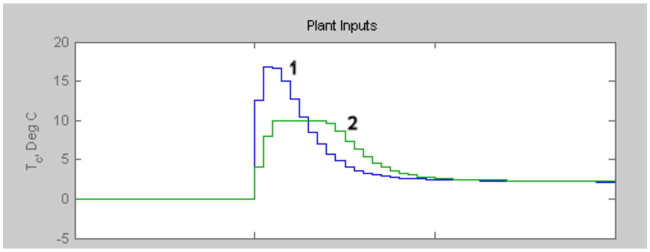
The larger control horizon and smaller control interval cause the unconstrained controller to make larger moves (see CSTR Manipulated Variable, Unconstrained (1) and MVconstraints (2), curve 1). The output settles at the new setpoint in about 5 time units rather than the 10 required previously (compare curve 1 in CSTR Outputs, Unconstrained (1) and MVconstraints (2) to curve 1 in Blocking Comparison, Outputs).

CSTR Manipulated Variable, Unconstrained (1) and MVconstraints (2) (curve 2) shows that the **Max Up Rate** constraint limits the size of the first two moves to 4 degrees. The third move hits the **Maximum** constraint at 10 degrees. The coolant temperature remains saturated at its upper limit for the next 7 control intervals, then slowly moves back down to its final value.

CSTR Outputs, Unconstrained (1) and MVconstraints (2) (curve 2) shows that the output response is slower, but still settles at the new setpoint smoothly within about 5 time units. This demonstrates the *anti-windup* protection provided automatically by the Model Predictive Control Toolbox controller.



CSTR Outputs, Unconstrained (1) and MVconstraints (2)



CSTR Manipulated Variable, Unconstrained (1) and MVconstraints (2)

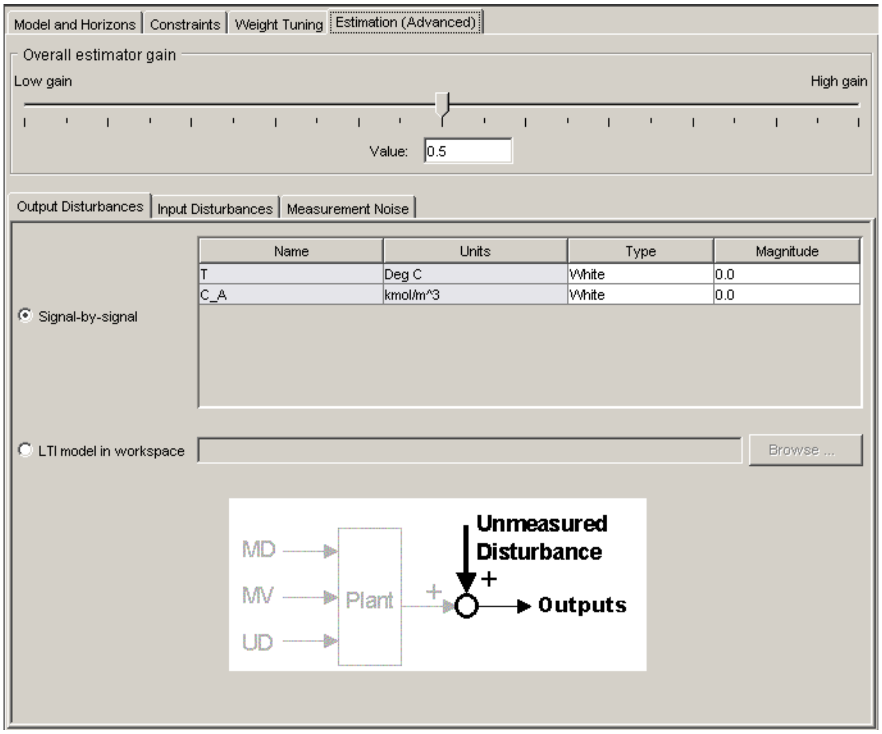
Disturbance Modeling and Estimation

The previous sections tested the controller's response to setpoint changes. In process control, disturbances rejection is often more important. The Model Predictive Control Toolbox product allows you to tailor the controller's disturbance response. The following example assumes that you have just completed the previous section, and the design tool is still open.

Select the first controller in the tree and rename it **InputSteps**. (Its settings should be identical to the **Unconstrained** controller of the previous section.)

Copy this controller. Rename the copy **OutputSteps**. Click its **Estimation** tab. The initial view should be as in Default Output Disturbance Settings for CSTR. Note the following:

- Model Predictive Control Toolbox default settings are being used. (The **Use Model Predictive Control Defaults** button restores these settings if you modify them.)
- The **Output Disturbances** tab is selected, and the **Signal-by-signal** option is selected. The graphic shows that the output disturbances add to each output.
- The tabular entries show, however, that these disturbance magnitudes are currently zero.



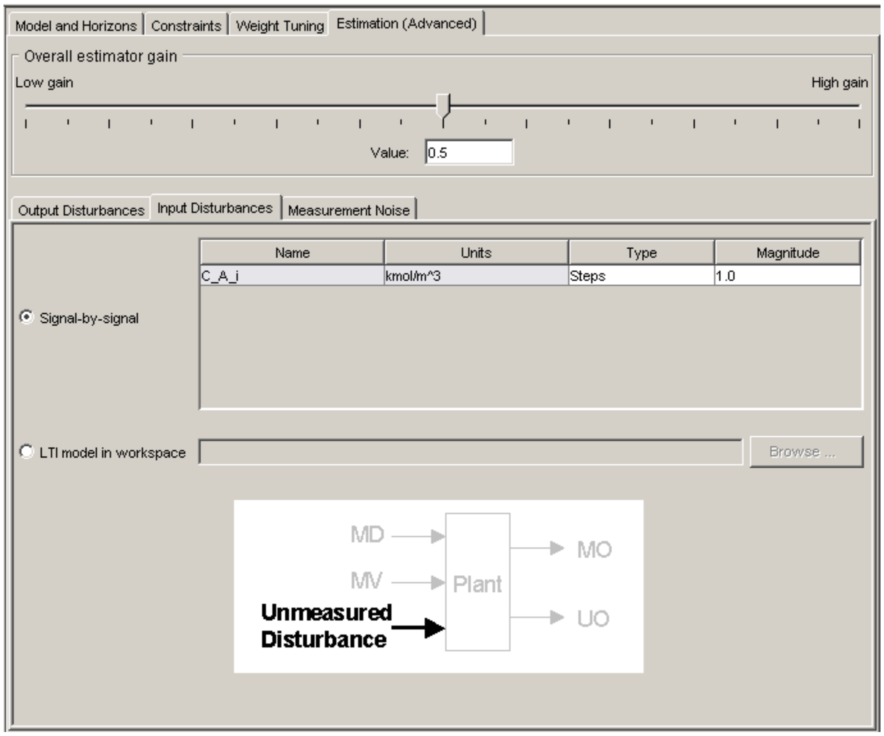
Value: 0.5

[✓ Signal-by-signal] LTI model in workspace

Name	Units	Type	Magnitude
T	Deg C	White	0.0
C_A	kmol/m^3	White	0.0

Default Output Disturbance Settings for CSTR

Click the **Input Disturbances** tab. (This would be inactive if the plant model had no unmeasured disturbances.) The view should change to that shown in Default Input Disturbance Settings for CSTR.



Value: 0.5

(●) Signal-by-signal

Name	Units	Type	Magnitude
C_A_i	kmol/m^3	Steps	1.0

Default Input Disturbance Settings for CSTR

In this case the disturbance magnitude is nonzero, and the disturbance type is `Steps`. Thus, the controller assumes that disturbances enter as random steps (integrated white noise) at the plant's unmeasured disturbance input.

Click the **Measurement Noise** tab, verifying that the controller is assuming white noise, magnitude 1 (not shown).

The following summarizes Model Predictive Control Toolbox default disturbance modeling assumptions for this case:

- Additive output disturbances: none
- Unmeasured input disturbances: random steps (integrated white noise)
- Measurement noise: white

In general, if your plant model includes unmeasured disturbance inputs, the toolbox default strategy will assume that they are dominant and sustained, as in the above example. This forces the controller to include an integrating mode, intended to eliminate steady-state error.

If the plant model contains no unmeasured input disturbances, the toolbox assumes sustained (integrated white noise) disturbances at the measured plant outputs.

If there are more measured outputs than unmeasured input disturbances, it assumes sustained disturbances in both locations according to an algorithm described in the product's online documentation.

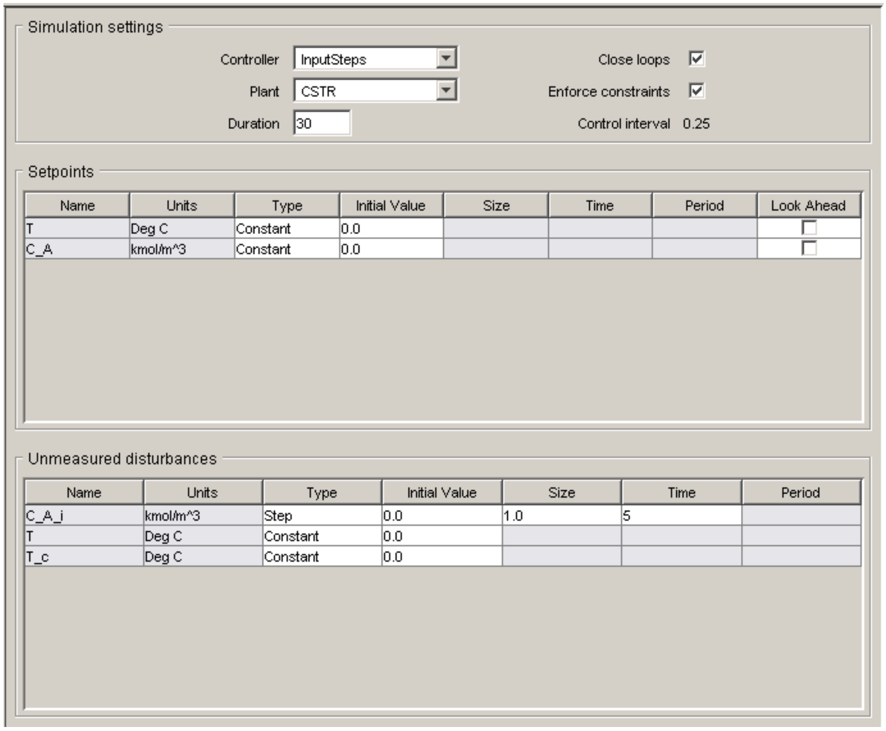
In any case, the design tool displays the assumptions being used.

To modify the estimation strategy in the `OutputSteps` controller, do the following:

- Click the **Input Disturbances** tab. Set the disturbance **Type** to `White`, and its **Magnitude** to `0`.
- Click the **Output Disturbances** tab. For the `T` output, set the disturbance Type to `Steps`, and its magnitude to `1`.

This *reverses* the default assumptions, i.e., the **OutputSteps** controller assumes that sustained disturbances enter at the plant output rather than at the unmeasured disturbance input. The **InputSteps** controller is still using the original (default) assumptions.

Next, select the first scenario in the tree. Rename it **Disturbance 1**, set its **Duration** to `30`, define all setpoints as constant zero values, and define a unit-step disturbance in the unmeasured input, `C_A_i`. If necessary, set its **Controller** option to `InputSteps` . CSTR Disturbance 1 Scenario shows the final **Disturbance 1** scenario.



Simulation settings

Controller	InputSteps	Close loops	✓
Plant	CSTR	Enforce constraints	✓
Duration	30	Control interval	0.25

Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
T	Deg C	Constant	0.0				☐
C_A	kmol/m^3	Constant	0.0				☐

Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Step	0.0	1.0	5	
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

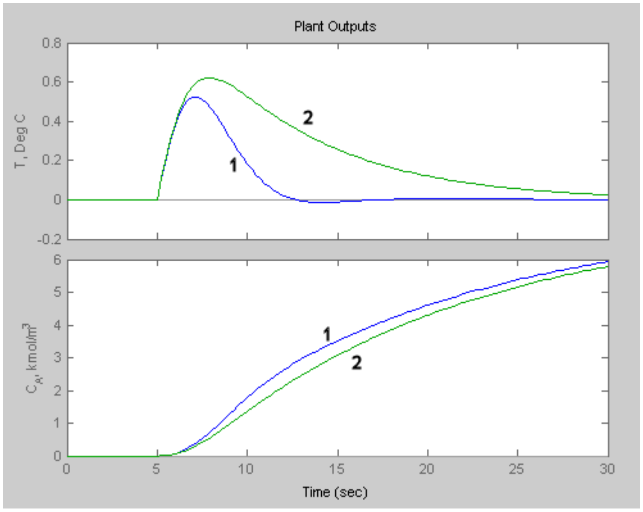
CSTR Disturbance 1 Scenario

Copy **Disturbance 1**. Rename the copy **Disturbance 2**, and set its **Controller** option to `OutputSteps`.

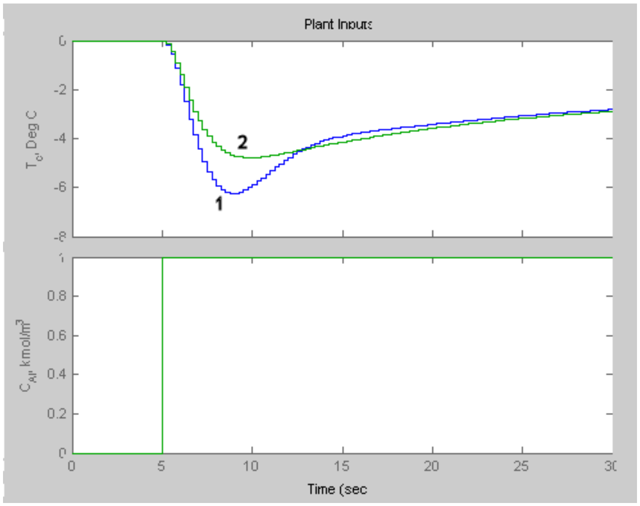
If necessary, close any open simulation plot windows. Simulate both scenarios. CSTR Outputs for Disturbance Scenarios 1 and 2 and CSTR Inputs for Disturbance Scenarios 1 and 2 show the results.

CSTR Outputs for Disturbance Scenarios 1 and 2 shows that default controller (case 1) returns to the setpoint in less than one third the time required by the modified controller (case 2). Its maximum deviation from the setpoint is also 10% smaller. CSTR Inputs for Disturbance Scenarios 1 and 2 shows that in both cases the input moves are smooth and of reasonable magnitude. (It also shows the input disturbance.)

The default controller expects unmeasured disturbances to enter as defined in the scenarios, so it's not surprising that the default controller performs better. The point is that the difference can be large, so it merits design consideration.

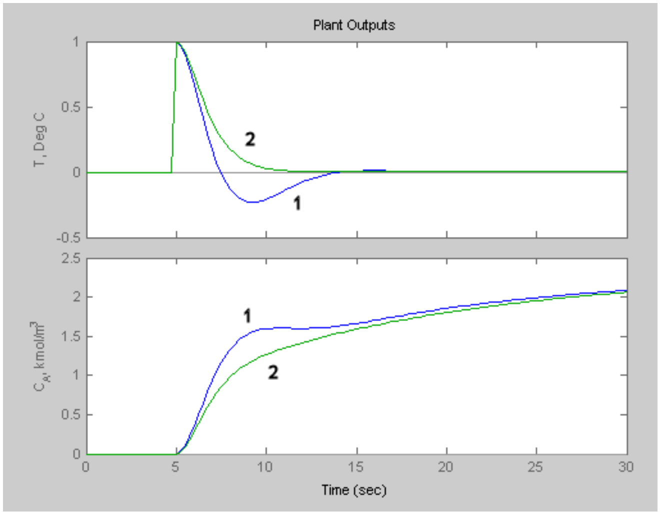


CSTR Outputs for Disturbance Scenarios 1 and 2



CSTR Inputs for Disturbance Scenarios 1 and 2

For comparison, reset the two scenarios so that the only disturbance is a one-degree step increase added to the measured reactor temperature. The modified controller (case 2) is designed for such disturbances, and CSTR Outputs, Output Disturbance Scenarios 1 and 2 shows that it performs better, but the difference is less dramatic than in the previous scenario. The default controller is likely to be best if the real process has multiple dominant disturbance sources.



CSTR Outputs, Output Disturbance Scenarios 1 and 2

Define Soft Output Constraints

The discussion in “Weight Tuning” on page 3-23, defined temperature control as the primary goal for the `CSTR` application. The predicted (but unmeasured) reactant concentration, C_A, could vary freely.

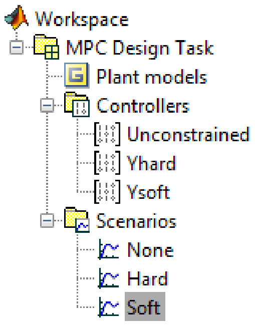
Suppose this were acceptable provided that C_A stayed below a specified maximum (above which unwanted reactions would occur). You can use an *output constraint* to enforce this specification.

Start with a single controller identical to the **InputSteps** controller described in “Disturbance Modeling and Estimation” on page 3-33. Rename it **Unconstrained**.

Right-click **Unconstrained** in the tree and select **Copy**. Rename the copy **Yhard**. Make another copy, naming it **Ysoft**.

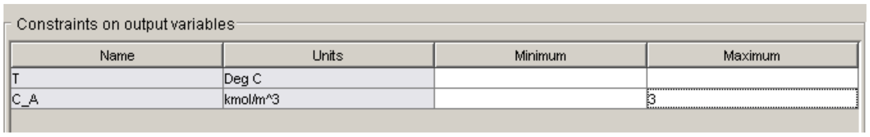
Similarly, start with a single scenario identical to CSTR Disturbance 1 Scenario, except that its **Controller** setting should be `Unconstrained`. Name this scenario **None**.

Right-click **None** in the tree and select **Copy**. Rename the copy **Hard**, and change its **Controller** setting to `Yhard`. Make another copy, naming it **Soft** and changing its **Controller** setting to `Ysoft`. Your tree should be as shown below.



- Workspace
  - MPC Design Task
    - Plant models
    - Controllers
      - Unconstrained
      - Yhard
      - Ysoft
    - Scenarios
      - None
      - Hard
      - Soft

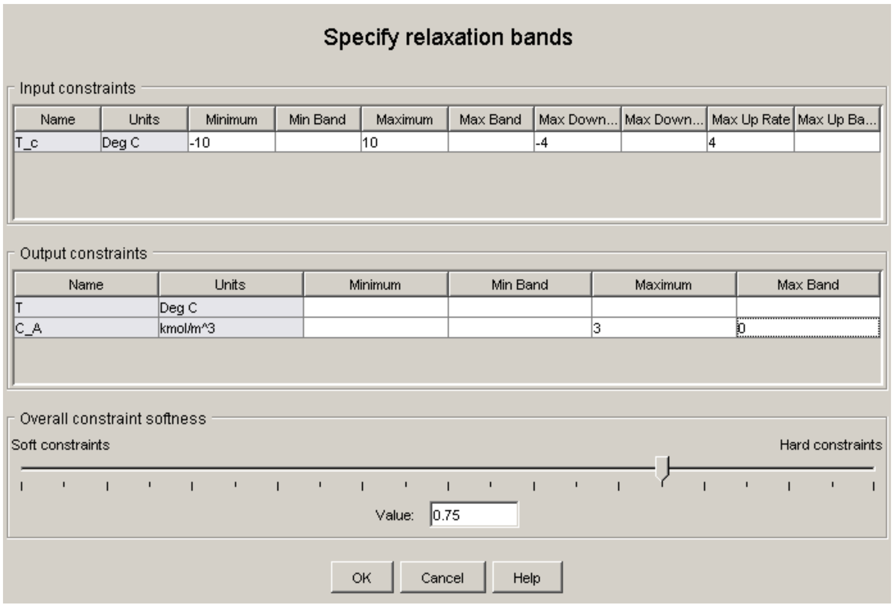
Select your **Yhard** controller. On the **Constraints** tab, set the maximum for C_A to 3 as shown below.



Constraints on output variables

Name	Units	Minimum	Maximum
T	Deg C		
C_A	kmol/m^3		3

Click the **Constraint Softening** button to open the dialog box in Constraint Softening Dialog Box. The **Input constraints** section shows the bounds on the inputs and their rates, and *relaxation bands*, which let you *soften* these constraints. By default, input constraints are *hard*, meaning that the controller tries to prevent any violation.



Input constraints

Name	Units	Minimum	Min Band	Maximum	Max Band	Max Down...	Max Down...	Max Up Rate	Max Up Ba...
T_c	Deg C	-10		10		-4		4	

Output constraints

Name	Units	Minimum	Min Band	Maximum	Max Band
T	Deg C				
C_A	kmol/m^3			3	0

Overall constraint softness

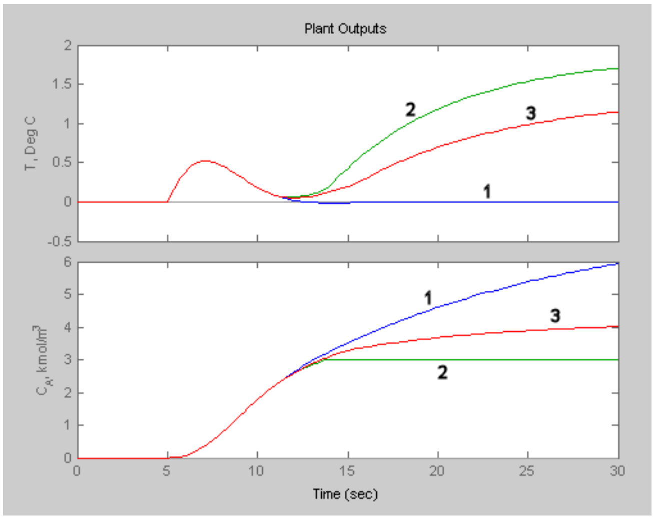
Value: 0.75

Constraint Softening Dialog Box

The **Output constraints** section lists the output limits and their relaxation bands. By default, the output constraints are *soft*. Make the C_A upper limit hard by entering a zero as its relaxation band (as in Constraint Softening Dialog Box).

Select the **Ysoft** controller. Define a soft upper bound on C_A by using the same settings shown in Constraint Softening Dialog Box, but with a relaxation band of 100 instead of 0.

Simulate the three scenarios in the order they appear in the tree, i.e., **None**, **Hard**, **Soft**. The resulting output responses appear in Constraint Softening Scenarios: 1 = None, 2 = Hard, 3 = Soft.



Constraint Softening Scenarios: 1 = None, 2 = Hard, 3 = Soft

Curve 1 is without output constraints, which is identical to curve 1 in CSTR Outputs for Disturbance Scenarios 1 and 2. This controller allows the C_A output to vary freely, but the controlled temperature returns to its setpoint within 10 time units after the disturbance happens.

Curve 2 shows the behavior with a hard upper limit at $C_A = 3$. Once C_A reaches this bound, the controller must use its one manipulated variable (T_c) to satisfy the constraint, so it's no longer able to control the temperature.

Curve 3 shows the result for a soft constraint. The controller reaches a compromise between the competing objectives: temperature control and constraint satisfaction. As you'd expect, performance lies between the curve 1 and curve 2 extremes.

The numerical value of the relaxation band represents a *relative tolerance* for constraint violations, not a strict limit (if it were the latter, it would be a hard constraint). If you were to increase its relaxation band (currently set at 100), performance would move toward Case 1, and vice versa.

If you have multiple constraints, you can harden or soften them simultaneously using the slider at the bottom of the controller's constraint softening dialog box (see Constraint Softening Dialog Box).

In general, you'll have to experiment to determine the settings that provide appropriate trade-offs for your application. In particular, the relaxation band settings interact with those on the controller's **Weight Tuning** tab (see “Weight Tuning” on page 3-23).

Another important factor is the expected numerical range for each variable. For example, if a particular variable stays within ±0.1 of its nominal value, it should have a small relaxation band relative to another variable having a range of ±100.

For details on the Model Predictive Control Toolbox constraint softening formulation, see ““Optimization Problem””.

Save Your Work

You'll usually want to save your design so you can reuse or revise it. You can save individual controllers or an entire project.

When you close the design tool, you'll be prompted to save new or modified designs. You can also save manually to preserve an intermediate state or guard against an unexpected shutdown.

This section covers the following topics:

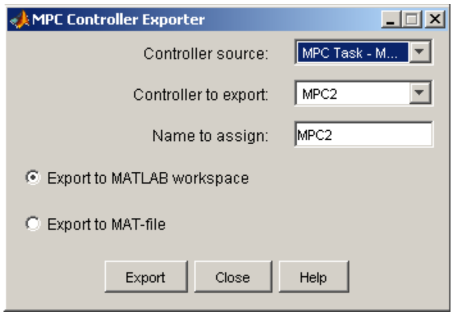
- “Exporting a Controller” on page 3-43
- “Saving a Project” on page 3-44

Exporting a Controller

To save a controller, *export* it to your MATLAB workspace or to a MAT-file. The former allows you to use the exported controller in command-line functions or a Simulink block.

Note Your workspace disappears when you exit MATLAB. A MAT-file is permanent, and you can reload it in a subsequent MATLAB session.

The following example assumes that the design tool is open and in the state described in the previous section. Suppose you want to export the controller to your workspace. Expand the tree if necessary, right-click **MPC2**, and select **Export Controller** from the resulting menu. The following dialog box appears.



Controller source: MPC Task - M...
Controller to export: MPC2
Name to assign: MPC2
[✓ Export to MATLAB workspace]
Export to MAT-file

The default behavior is to export the selected controller to the workspace. Click **Export** to confirm. You can verify the export by typing

```
whos
```

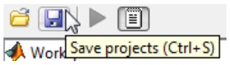
at the MATLAB prompt. The resulting list should include an `mpc` object named `MPC2`. Type

```
MPC2
```

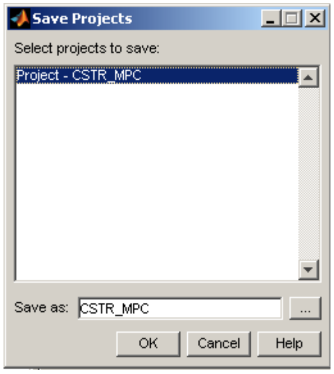
to display the object's properties.

Saving a Project

To save your entire project, click the toolbar's **Save** button.



The following dialog box appears.



Dialog Box for Saving a Controller Design Project

The default behavior saves the current project (named `Project - CSTR_MPC` in this case) in a MAT-file (called `CSTR_MPC` here). If the design tool had contained other projects, they would appear in the list, and you could select the ones you wanted to save.

The MAT-file will be saved in the default folder. To verify the location, click the **Browse (...)** button and change the folder if necessary. When ready to save, click **OK**.

Load Your Saved Work

The following assumes that you've saved a project as described in the previous section. To reload this project, close the design tool if it's open. Also clear any `mpc` objects from your workspace. (Type `whos` at the MATLAB prompt for a list of objects.) For example, if `MPC1` and `MPC2` are in your workspace, type

```
clear MPC1 MPC2
```

to clear (remove) them.

If you've closed the `CSTR_MPC` model, open it. Double-click the MPC Controller block to open its mask, and verify that the **MPC Controller** parameter is set to `MPC2`.

Note If you were to attempt to run the `CSTR_MPC` model at this stage, an error dialog box would indicate that the MPC Controller block was unable to initialize. The `MPC2` object specified in the block mask must be loaded into your workspace or be part of an active design tool task.

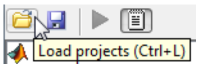
You could define the required `MPC2` object in one of the following ways:

- Import `MPC2` from a MAT-file (assuming you had saved it as explained in "Exporting a Controller" on page 3-43).
- Load the model's project file, which contains a copy of `MPC2`.

To use the second approach, open the design tool by typing

```
mpctool
```

in the MATLAB Command Window. This creates a blank Model Predictive Control Toolbox project called **MPC Design Task**. Click **Load** on the toolbar.



This opens a dialog box similar to that shown in Dialog Box for Saving a Controller Design Project. Use it to select the project file you've saved, and then click **OK** to load the project. It should appear in the tree. Verify that it contains a controller named `MPC2`.

Run the `CSTR_MPC` model in Simulink. The block mask automatically retrieves **MPC2** from the design tool, and the simulation runs. In other words, loading the project automatically restores the link between the design tool and the MPC Controller block.

Test Controller Robustness

It's good practice to test your controller's sensitivity to prediction errors. Classical phase and gain margins are one way to quantify robustness for a SISO application. Robust Control Toolbox™ software provides sophisticated approaches for MIMO systems. It can also be helpful to run simulations. The following example illustrates the simulation approach.

Plant Model Perturbation

Create a perturbed version of a model.

For this example, use the nonlinear `CSTR` model discussed in "CSTR Model".

```
CSTRp = CSTR;
CSTRp.a=[-0.0303   -0.0113
         -0.0569   -0.1836];
CSTRp.b=[-0.0857    0.0191
          0.1393    0.4241];
```

This creates a copy of `CSTR` called `CSTRp`, then replaces the state space *A* and *B* matrices with perturbed versions (compare to the originals defined in "State-Space Models" on page 2-10). Use the following command to compare the two step responses:

```
step(CSTR, CSTRp)
```

Observe the difference in the responses (not shown).

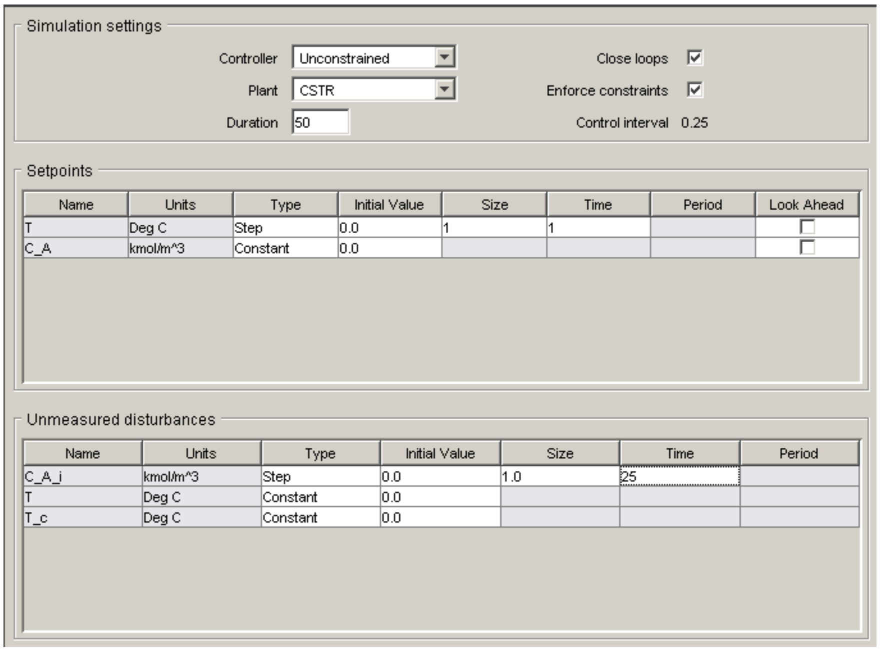
Select **Plant models** in the tree. Click the **Import** button and import the `CSTRp` model.

Simulation Tests

Delete all controllers except the first one in the tree. If necessary, make its settings identical to **Unconstrained** (see "Defining Manipulated Variable Constraints" on page 3-31).

Delete all scenarios except the first, naming that **Accurate Model**. Define its properties as shown in Robustness Test, Accurate Plant Model Scenario. The scenario begins with a step change in the temperature setpoint, followed 25 time units later by a step disturbance in the reactant entering the `CSTR`.

Copy **Accurate Model**. Rename the copy **Perturbed Model**, and set its **Plant** option to `CSTRp`. Thus, both scenarios use the same controller, which is based on the `CSTR` model, but the **Perturbed Model** scenario uses a different model to represent the “real” plant. This tests the controller's robustness to a change in plant parameters.



Simulation settings

Controller	Unconstrained	Close loops	✓
Plant	CSTR	Enforce constraints	✓
Duration	50	Control interval	0.25

Setpoints

Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
T	Deg C	Step	0.0	1	1		☐
C_A	kmol/m^3	Constant	0.0				☐

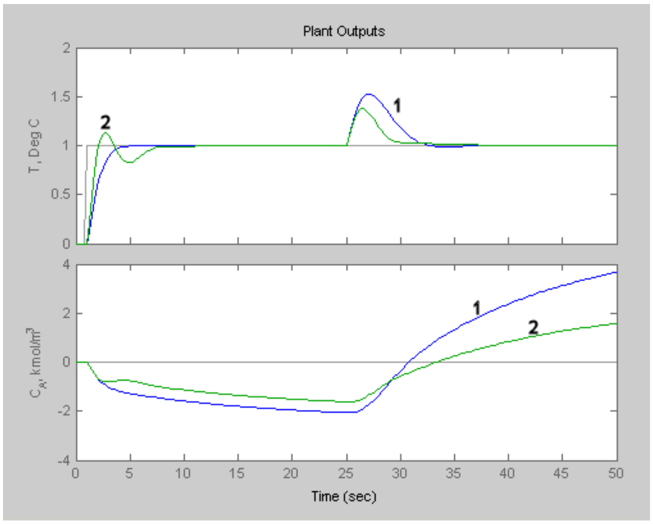
Unmeasured disturbances

Name	Units	Type	Initial Value	Size	Time	Period
C_A_i	kmol/m^3	Step	0.0	1.0	25	
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

Robustness Test, Accurate Plant Model Scenario

Simulate the two scenarios. Robustness Test, Accurate Model (1) and Perturbed Model (2) shows the output responses. As expected, setpoint tracking degrades when the model is inaccurate, but performance is still acceptable.

The disturbance rejection appears to *improve* with the perturbed model. This is a consequence of the perturbations used. The gain for the T/C_{Ai} output/input pair is about 15% smaller in the `CSTRp` model, which has two beneficial effects: the actual impact of the disturbance is reduced, and the controller is aggressive because it expects a larger impact.



Robustness Test, Accurate Model (1) and Perturbed Model (2)

Note MIMO applications are usually more sensitive to model error than SISO applications, so robustness testing is especially recommended for MIMO cases.

Design Controller for Plant with Delays

This example shows how to design an MPC controller for a plant with delays.

The Model Predictive Control Toolbox can handle models that include delays. A typical example is the distillation column model, which includes a delay in each input/output channel. The presence of delays influences controller performance, and your controller specifications should account for them.

Create the distillation column plant model.

```
g11 = tf( 12.8, [16.7 1], 'IOdelay', 1.0,'TimeUnit','minutes');
g12 = tf(-18.9, [21.0 1], 'IOdelay', 3.0,'TimeUnit','minutes');
g13 = tf(  3.8, [14.9 1], 'IOdelay', 8.1,'TimeUnit','minutes');
g21 = tf(  6.6, [10.9 1], 'IOdelay', 7.0,'TimeUnit','minutes');
g22 = tf(-19.4, [14.4 1], 'IOdelay', 3.0,'TimeUnit','minutes');
g23 = tf(  4.9, [13.2 1], 'IOdelay', 3.4,'TimeUnit','minutes');
DC = [g11 g12 g13
      g21 g22 g23];
DC.InputName = {'Reflux Rate', 'Steam Rate', 'Feed Rate'};
DC.OutputName = {'Distillate Purity', 'Bottoms Purity'};
DC = setmpcsignals(DC, 'MD', 3);
```

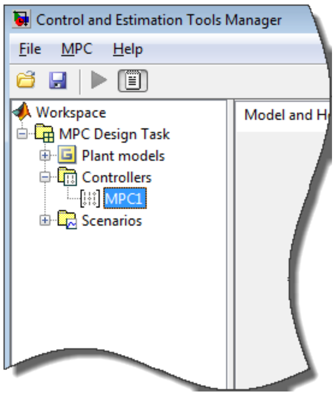
The largest `ioDelay` of the `DC` model delay is 8.1 minutes.

Open the Model Predictive Control Toolbox design tool.

```
mpctool
```

Click **Import Plant** and import the `DC` model into the tool.

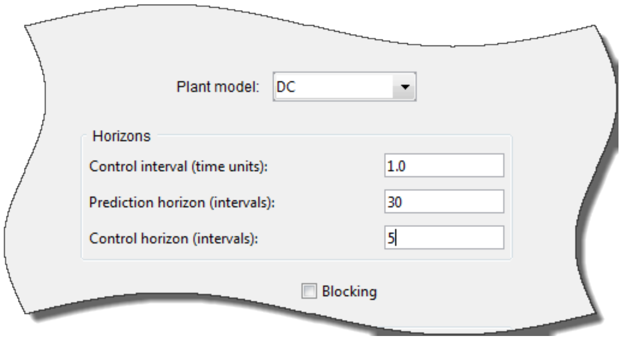
In the tree, select **MPC Design Task** > **Controllers** > **MPC1**.



It is good practice to specify the prediction and control horizons such that

$$P - M \gg t_{d,max} / \Delta t$$

Here, P is the prediction horizon, M is the control horizon, $t_{d,max}$ is the maximum delay, and Δt is the control interval. In the **Control interval (time units)** box, which specifies Δt, enter `1`. Then, because the maximum `ioDelay` for the `DC` model is 8.1 minutes, $t_{d,max}/\Delta t$ is 8.1. You can satisfy the inequality by setting `P = 30` and `M = 5`. In the **Prediction horizon (intervals)** box, enter `30`, and in the **Control horizon (intervals)** box, enter `5`.


Plant model: DC

Horizons

Control interval (time units): 1.0

Prediction horizon (intervals): 30

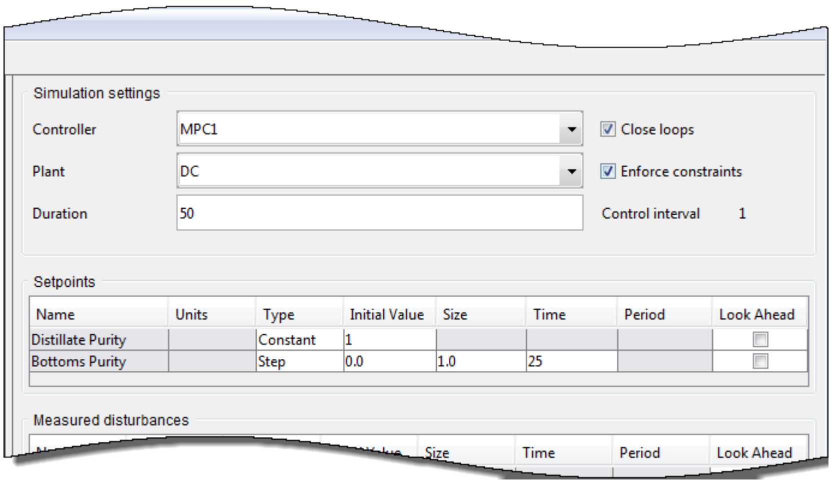
Control horizon (intervals): 5

Blocking

Now that you have specified the controller horizons, simulate the model to test the controller performance. In the tree, select **Scenarios** > **Scenario1**.

Specify the simulation settings as follows:

- In the **Duration** box, enter `50`.
- In the **Setpoints** table, for the **Distillate Purity** setpoint, enter `1` in the **Initial Value** box.
- In the **Setpoints** table, for the **Bottoms Purity** setpoint, from the **Type** list, select `Step`. In the **Time** box, enter `25`.



Simulation settings

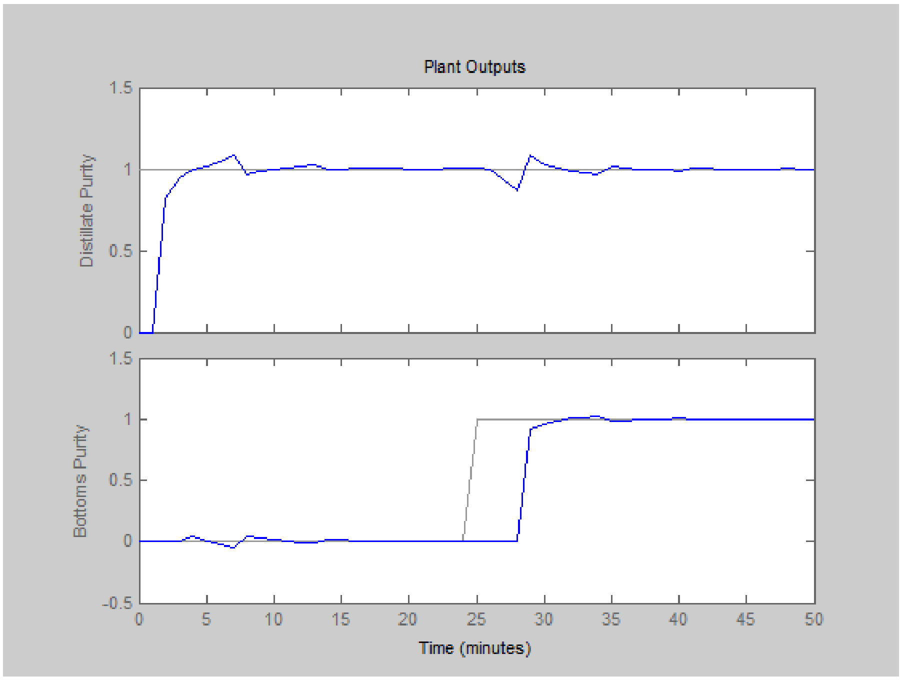
Controller	MPC1	✓ Close loops	
Plant	DC	✓ Enforce constraints	
Duration	50	Control interval	1

Setpoints

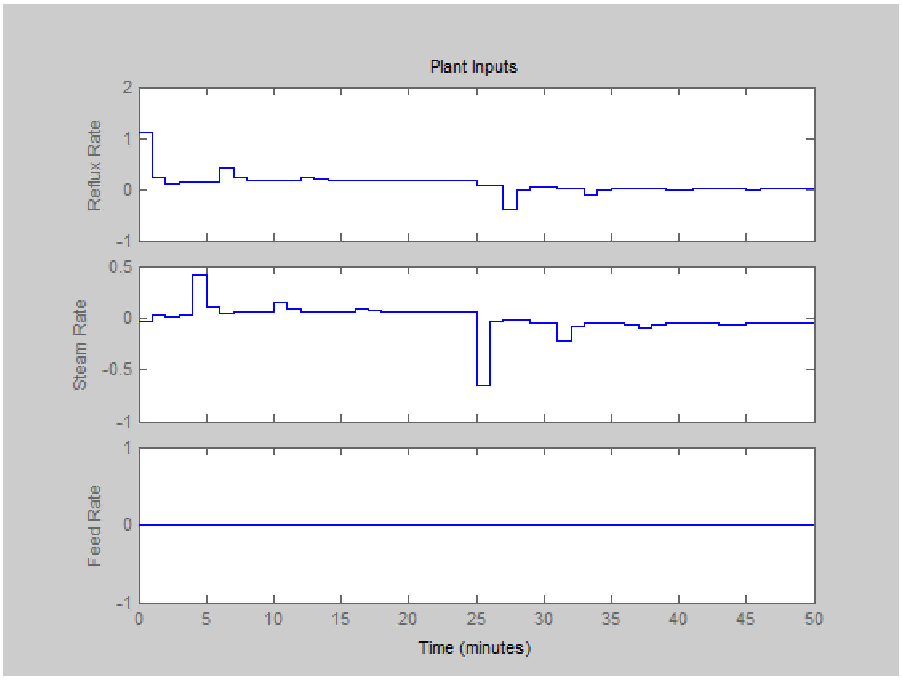
Name	Units	Type	Initial Value	Size	Time	Period	Look Ahead
Distillate Purity		Constant	1				☐
Bottoms Purity		Step	0.0	1.0	25		☐

Measured disturbances

Click **Simulate**.



The plant outputs plot shows that the first output does not respond for the first minute, which corresponds to the delay from the first input. The first output reaches the setpoint in two minutes and settles quickly. Similarly, the second output does respond for three minutes, which corresponds to the delay from the second input, and settles rapidly afterwards. Changing the setpoint for the Bottoms Purity output disturbs the Distillate Purity output, but the magnitude of this interaction is less than 10%.

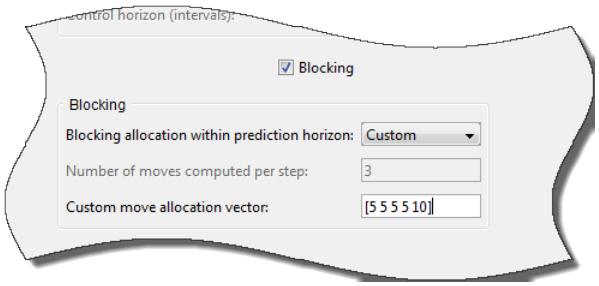


The plant inputs plot shows that the initial input moves are more than five times the final change. Also, there are periodic pulses in the control action as the controller attempts to counteract the delayed effects of each input on the two outputs.

You can moderate these effects by tuning the controller weights or by specifying a custom blocking strategy. For this example, use the latter approach. In the tree, select **MPC Design Task** > **Controllers** > **MPC1**. Select the **Blocking** check box.

In the **Blocking** section:

- From the **Blocking allocation within prediction horizon** list, select `Custom`
- In the **Custom move allocation vector**, enter `[5 5 5 5 10]`



Control horizon (intervals):

☑ Blocking

Blocking

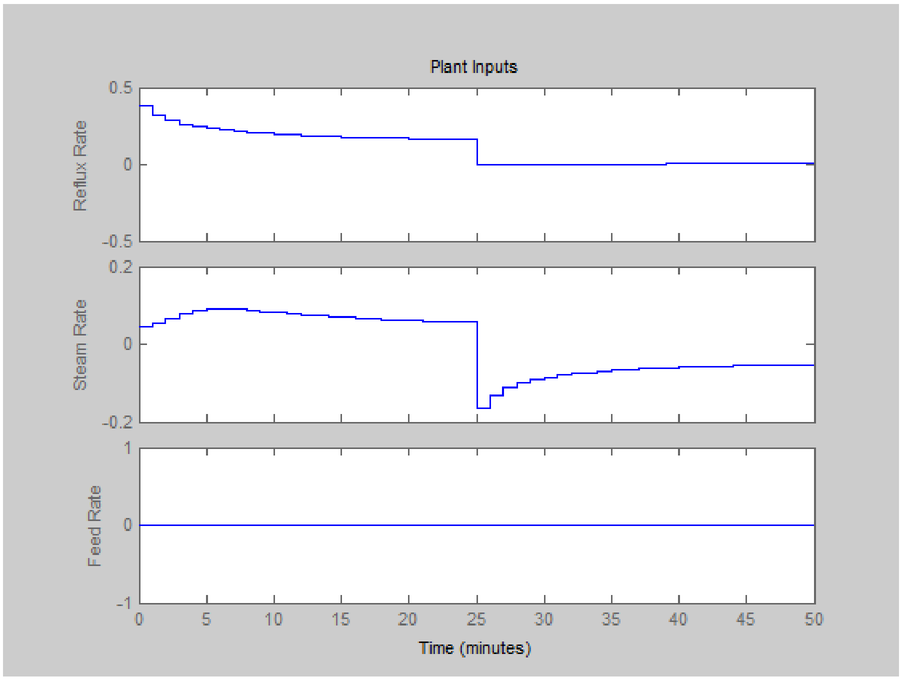
Blocking allocation within prediction horizon: Custom

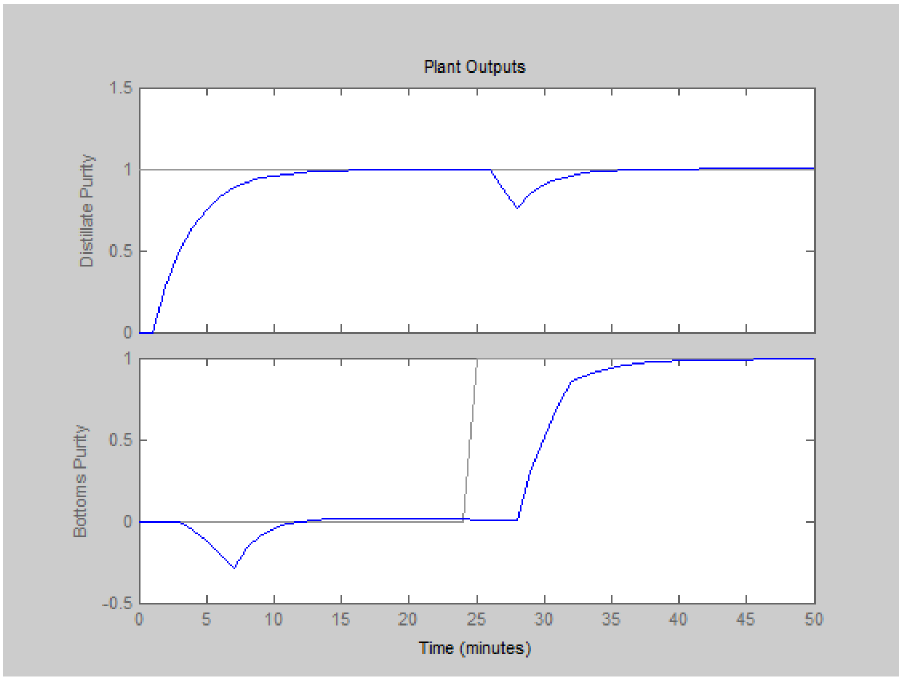
Number of moves computed per step: 3

Custom move allocation vector: [5 5 5 5 10]

In the tree, select **Scenarios** > **Scenario1**.

Click **Simulate**.





The initial input moves are much smaller and the moves are less oscillatory. The trade-off is a slower output response, with about 20% interaction between the outputs.

Related Examples

- “Specify Multi-Input Multi-Output (MIMO) Plants” on page 2-16

Design Controller for Nonsquare Plant

A *nonsquare* plant has an unequal number of manipulated variables and output variables. This is common in practice, and the Model Predictive Control Toolbox software supports an excess of manipulated variables or outputs. In such cases you will usually need to modify default toolbox settings.

This section covers the following topics:

- "More Outputs Than Manipulated Variables" on page 3-57
- "More Manipulated Variables Than Outputs" on page 3-58

More Outputs Than Manipulated Variables

When there are excess outputs, you can't hold each at a setpoint. You have the following options:

- Enforce setpoints on all outputs, in which case all will deviate from their setpoints to some extent.
- Specify that certain outputs need not be held at setpoints by setting their weights to zero on the controller's **Weight Tuning** tab.

The initial test of the `CSTR` controller used option 1 (the default), which caused both outputs to deviate from their setpoints (see Plant Outputs for T Setpoint Scenario with Added Data Markers). You can adjust the offset in each output by changing the output weights. Increasing an output weight decreases the offset in that output (at the expense of increased offset in other outputs).

The modified `CSTR` controller used option 2 (see the discussion in "Weight Tuning" on page 3-23). In general, if the application has N_e more outputs than manipulated variables, setting N_e output weights to zero should allow the remaining outputs to be held at setpoints (unless the manipulated variables are constrained). This was the case for the modified `CSTR` controller (see Improved Setpoint Tracking for CSTR Temperature).

Outputs that have been "sacrificed" by setting their weights to zero can still be useful. If measured, they can help the controller to estimate the plant's state, thereby improving its predictions. They can also be used as indicators, or as variables to be held within an operating region defined by output constraints.

More Manipulated Variables Than Outputs

In this situation, default Model Predictive Control Toolbox settings should provide offset-free output-setpoint tracking, but the manipulated variables are likely to drift.

One way to avoid this is to use *manipulated variable setpoints*. If there are N_e excess manipulated variables and you hold N_e of them at target values, the rest should not drift. Rather, they will attain the values needed to eliminate output offset.

To define a manipulated variable setpoint:

1. Enter the setpoint value in the **Nominal** field in the signal properties view – see Model Predictive Control Toolbox Design Tool's Signal Definition View.
2. Assign a nonzero *input weight* using the **Weight** entry on the controller's **Weight Tuning** tab – see Controller Options — Weight Tuning Tab.

In step 2, the magnitude of the input weight determines the extent to which the manipulated variable can deviate from its target during a transient. See “Input Weights” on page 3-27 for more discussion and mathematical details.

You might want to allow such deviations temporarily in order to provide better output setpoint tracking. In that case, use a relatively small input weight. If you want the manipulated variable to stay near its target value at all times, increase its input weight.

Another way to avoid drift is to constrain one or more manipulated variables to a narrow operating region. You can even hold an MV constant by setting its lower and upper bounds to the same value (in which case its nominal value should also be set to this value), or by setting both of its rate constraints to zero. To define constraints, use the controller's **Constraints** tab (see “Defining Manipulated Variable Constraints” on page 3-31).

Designing Controllers Using the Command Line

Design Controller at the Command Line

In this section...

"Design Controller Using the Design Tool" on page 3-2 shows how to use the Model Predictive Control Toolbox design tool to create a controller and test it. You might prefer to use functions instead. They allow access to options not available in the design tool, as well as automation of repetitive tasks using scripts and customized plotting.

Create a Controller Object

This topic uses the `CSTR` model described in "CSTR Model" on page 2-18 as an example. You must have the linear plant model `CSTR` in the base workspace before beginning this tutorial.

Use the `mpc` function to create a controller. For example, type

```
Ts = 1;
MPCobj = mpc(CSTR,Ts);
```

to create one based on the `CSTR` model with a control interval of 1 time unit and all other parameters at their default values.

Note `MPCobj` is an MPC object. It contains a complete controller definition for use with Model Predictive Control Toolbox software.

To display the controller's properties in the Command Window, type

```
display(MPCobj)
```

or type the object's name without a trailing semicolon.

View and Alter Controller Properties

Once you've defined an MPC object, it's easy to alter its properties. For a description of the editable properties, type:

```
mpcprops
```

To display the list of properties and their current values, type

```
get(MPCobj)
```

```
   ManipulatedVariables (MV): [1x1 struct]
        OutputVariables (OV): [1x2 struct]
   DisturbanceVariables (DV): [1x1 struct]
                 Weights (W): [1x1 struct]
                       Model: [1x1 struct]
                          Ts: 1
                   Optimizer: [1x1 struct]
       PredictionHorizon (P): 10
              ControlHorizon: 2
                     History: [2e+003 7 21 20 18 20.1]
                       Notes: {}
                    UserData: []
```

(Your `History` entry will differ.) To alter one of these properties, you can use the syntax

```
ObjName.PropName = value;
```

where `ObjName` is the object name, and `PropName` is the property you want to set. For example, to change the prediction horizon from 10 (the default) to 15, type:

```
MPCobj.P = 15;
```

Note You can abbreviate property names provided that the abbreviation is unambiguous.

As shown above, many of the properties are MATLAB *structures* containing additional properties. For example, type

```
MPCobj.MV
```

```
            Min: -Inf
            Max: Inf
         MinECR: 0
         MaxECR: 0
        RateMin: -Inf
        RateMax: Inf
     RateMinECR: 0
     RateMaxECR: 0
         Target: 'nominal'
           Name: 'T_c'
          Units: ''
```

This shows that the default controller has no constraints on the manipulated variable. To include constraints as shown in Entering CSTR Manipulated Variable Constraints, enter:

```
MPCobj.MV.Min = -10;
MPCobj.MV.Max = 10;
MPCobj.MV.RateMin = -4;
MPCobj.MV.RateMax = 4;
```

There are two outputs in this case, so `MPCobj.OV` is a 1-by-2 structure. To set measurement units to the values shown in Controller Options — Weight Tuning Tab, enter:

```
MPCobj.Model.Plant.OutputUnit = {'Deg C','kmol/m^3'};
```

Note The unit properties for display purposes only and is optional.

Finally, check the default weights by typing

```
MPCobj.W

        ManipulatedVariables: 0
    ManipulatedVariablesRate: 0.1000
             OutputVariables: [1 1]
                         ECR: 100000
```

Change to the values shown in Controller Options — Weight Tuning Tab by typing:

```
MPCobj.W.ManipulatedVariablesRate = 0.3;
MPCobj.W.OutputVariables = [1 0];
```

You can also specify time-varying weights and constraints. The time-varying weights and constraints are defined for the prediction horizon, which shifts at each time step. This implies that as long as the property is not changed, the set of time-varying parameters is the same at each time step. To learn how to specify time-varying constraints and weights in the GUI, see ““Constraints Tab”” and ““Weight Tuning Tab””.

The time-varying weights modify the tuning of the unconstrained controller response. To specify a different weight for each step in the prediction horizon, modify the `Weight`property. For example,

```
MPCobj.W.OutputVariables = [0.1 0;0.2 0;0.5 0;1 0];
```

deemphasizes setpoint tracking errors early in the prediction horizon. The default weight of 1 is used for the fourth step and beyond.

Constraints can also be time varying. The time-varying constraints have a nonlinear effect when they are active. For example,

```
MPCobj.MV.RateMin = [-4;-3.5;-3;-2.5];
MPCobj.MV.RateMax = [4;3.5;3;2.5];
```

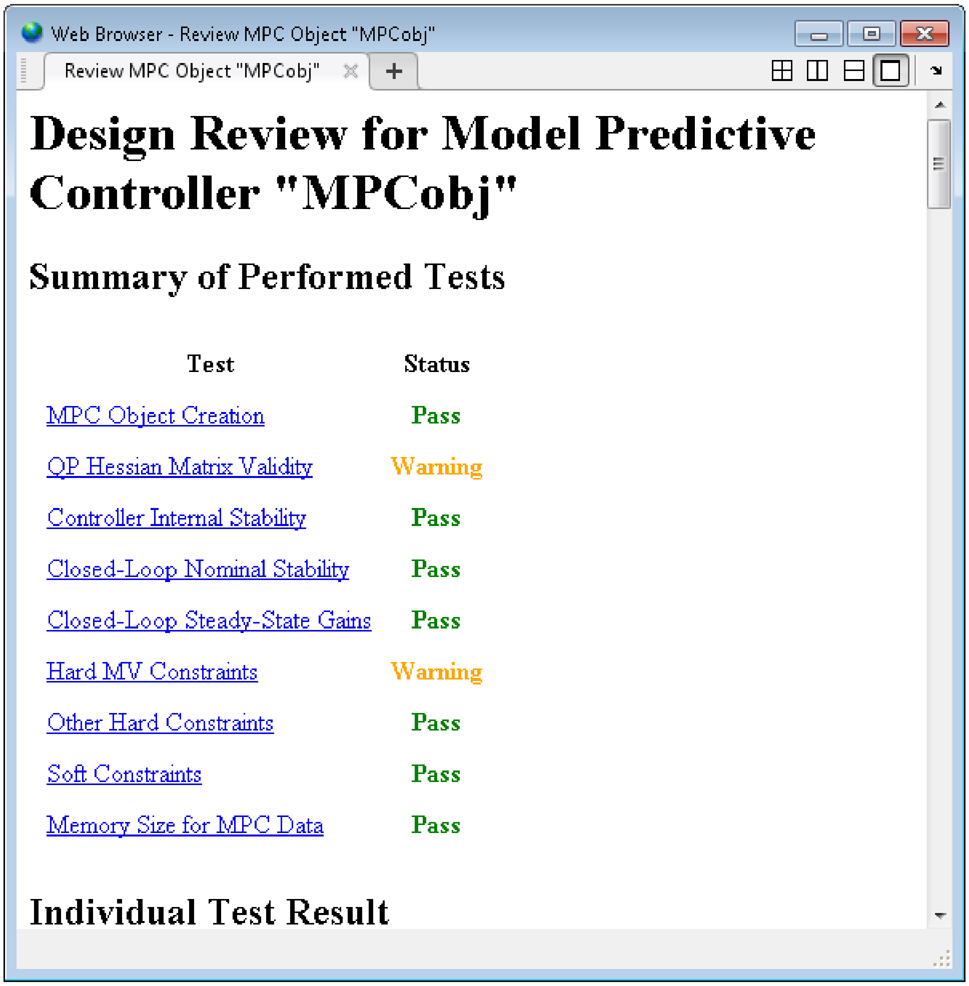
forces `MV` to change more and more slowly along the prediction horizon. The constraint of -2.5 and 2.5 is used for the fourth step and beyond.

You could also alter the controller's disturbance rejection characteristics using functions that parallel the design tool's disturbance modeling options (described in “Disturbance Modeling and Estimation” on page 3-33). See the reference pages for the `setEstimator`, `setindist`, and `setoutdist` functions.

Review Controller Design

Use the `review` command to examine the potential controller stability and performance issues at runtime. `review` generates a report on the results of the tests it performs.

```
review(MPCobj)
```


Web Browser - Review MPC Object "MPCobj"
Review MPC Object "MPCobj"

Design Review for Model Predictive Controller "MPCobj"

Summary of Performed Tests

Test	Status
MPC Object Creation	Pass
QP Hessian Matrix Validity	Warning
Controller Internal Stability	Pass
Closed-Loop Nominal Stability	Pass
Closed-Loop Steady-State Gains	Pass
Hard MV Constraints	Warning
Other Hard Constraints	Pass
Soft Constraints	Pass
Memory Size for MPC Data	Pass

Individual Test Result

In this example, the `review` command found two potential issues in this design. The first warning asks whether the user intends to have zero weight on the `C_A` output. The second warning asks the user to avoid having hard constraints on both `MV` and `MVRate`.

Perform Linear Simulations

To run a linear simulation, use the `sim` function. For example, given the `MPCobj` controller defined in the previous section, type:

```
T = 26;
r = [2 0];
sim(MPCobj,T,r);
```

This simulates the closed-loop response for a duration of 26 control intervals with a setpoint of 2 for the first output (the reactor temperature) and 0 for the second output (the residual concentration). Recall that the second output's tuning weight is zero (see the discussion in “Output Weights” on page 3-25), so its setpoint is ignored.

By default, the same linear model is used for controller predictions and the plant, i.e., there is no plant/model mismatch. You can alter this as shown in “Simulation Options” on page 4-7.

When you use the above syntax (no output variables), `sim` automatically plots the plant inputs and outputs (not shown, but see Improved Setpoint Tracking for CSTR Temperature and Plant Inputs for Modified Rate Weight for results of a similar scenario).

Simulation Options

You can modify simulation options using the `mpcsimopt` function. For example, the code

```
MPCopts = mpcsimopt;
MPCopts.Constraints = 'off';
sim(MPCobj,T,r,MPCopts)
```

runs an unconstrained simulation. Comparing to the case described earlier, the controller's first move now exceeds 4 units (the specified rate constraint).

Other options include the addition of a specified noise sequence to the manipulated variables or measured outputs, open-loop simulations, a look-ahead option for better setpoint tracking or measured disturbance rejection, and plant/model mismatch.

For example, the following code defines a new plant model having gains 50% larger than those in the `CSTR` model used in the controller, then repeats the above simulation:

```
Plant = 1.5*CSTR;
MPCopts.Model = Plant;
sim(MPCobj,T,r,MPCopts)
```

In this case, the plant/model mismatch degrades controller performance, but only slightly. Degradation can be severe and must be tested on a case-by-case basis.

Save Calculated Results

If you'd like to save simulation results in your workspace, use the following `sim` function format:

```
[y,t,u] = sim(MPCobj,T,r);
```

This suppresses automatic plotting, instead creating variables `y`, `t`, and `u`, which hold the computed outputs, time, and inputs, respectively. A typical use is to create customized plots. For example, to plot both outputs on the same axis versus time, you could type:

```
plot(t,y)
```

Simulate Controller with Nonlinear Plant

You can use `sim` to simulate a closed-loop system consisting of a linear plant model and an MPC controller.

If your plant is a nonlinear Simulink model, you must linearize the plant (see "Linearization Using Linear Analysis Tool in Simulink Control Design") and design a controller for the linear model (see "Design Controller in Simulink"). To simulate the system, specify the controller in the MPC block parameter **MPC Controller** field and run the closed-loop Simulink model.

Alternatively, your nonlinear model might be a MEX-file, or you might want to include features unavailable in the MPC block, such as a custom state estimator. The `mpcmove` function is the Model Predictive Control Toolbox computational engine, and you can use it in such cases. The disadvantage is that you must duplicate the infrastructure that the `sim` function and the MPC block provide automatically.

The rest of this section covers the following topics:

- "Nonlinear CSTR Application" on page 4-9
- "Example Code for Successive Linearization" on page 4-10
- "CSTR Results and Discussion" on page 4-12

Nonlinear CSTR Application

The `CSTR` model described in "Linearize Simulink Models" on page 2-20 is a strongly nonlinear system. As shown in "Design Controller in Simulink", a controller can regulate this plant, but degrades (and might even become unstable) if the operating point changes significantly.

The objective of this example is to redefine the predictive controller at the beginning of each control interval so that its predictive model, though linear, represents the latest plant conditions as accurately as possible. This will be done by linearizing the nonlinear model repeatedly, allowing the controller to adapt as plant conditions change. See references [1] and [2] for more details on this approach.

Example Code for Successive Linearization

In the following code, the simulation begins at the CSTR model's nominal operating point (concentration = 8.57) and moves to a low concentration (= 2) where the reaction rate is much higher. The required code is as follows:

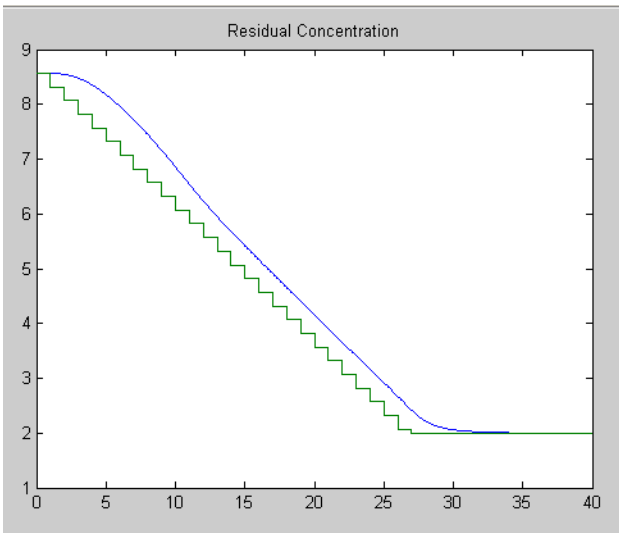
```
[sys, xp] = CSTR_INOUT([],[],[],'sizes');
up = [10 298.15 298.15];
u = up(3);
tsave = []; usave = []; ysave = []; rsave = [];
Ts = 1;
t = 0;
while t < 40
    yp = xp;
    % Linearize the plant model at the current conditions
    [a,b,c,d]=linmod('CSTR_INOUT', xp, up );
    Plant = ss(a,b,c,d);
    Plant.InputGroup.ManipulatedVariables = 3;
    Plant.InputGroup.UnmeasuredDisturbances = [1 2];
    Model.Plant = Plant;

    % Set nominal conditions to the latest values
    Model.Nominal.U = [0 0 u];
    Model.Nominal.X = xp;
    Model.Nominal.Y = yp;

    dt = 0.001;

    simOptions.StartTime = num2str(t);
    simOptions.StopTime = num2str(t+dt);
    simOptions.LoadInitialState = 'on';
    simOptions.InitialState = 'xp';
    simOptions.SaveTime = 'on';
    simOptions.SaveState = 'on';
    simOptions.LoadExternalInput = 'on';
    simOptions.ExternalInput = '[t up; t+dt up]';

    simOut = sim('CSTR_INOUT',simOptions);

    T = simOut.get('tout');
    XP = simOut.get('xout');
    YP = simOut.get('yout');

    Model.Nominal.DX = (1/dt)*(XP(end,:)' - xp(:));
    % Define MPC Toolbox controller for the latest model
    MPCobj = mpc(Model, Ts);
    MPCobj.W.Output = [0 1];

    % Ramp the setpoint
    r = max([8.57 - 0.25*t, 2]);

    % Compute the control action
    if t <= 0
        xd = [0; 0];
        x = mpcstate(MPCobj, xp, xd, [], u);
    end

    u = mpcmove(MPCobj, x, yp, [0 r], []);

    % Simulate the plant for one control interval
    up(3) = u;

    simOptions.StartTime = num2str(t);
    simOptions.StopTime = num2str(t+Ts);
    simOptions.InitialState = 'xp';
    simOptions.ExternalInput = '[t up; t+Ts up]';

    simOut = sim('CSTR_INOUT',simOptions);

    T = simOut.get('tout');
    XP = simOut.get('xout');
    YP = simOut.get('yout');

    % Save results for plotting
    tsave = [tsave; T];
    ysave = [ysave; YP];
    usave = [usave; up(ones(length(T),1),:)];
    rsave = [rsave; r(ones(length(T),1),:)];

    xp = XP(end,:)';

    t = t + Ts;
end

figure(1)
plot(tsave,[ysave(:,2) rsave])
title('Residual Concentration')
```

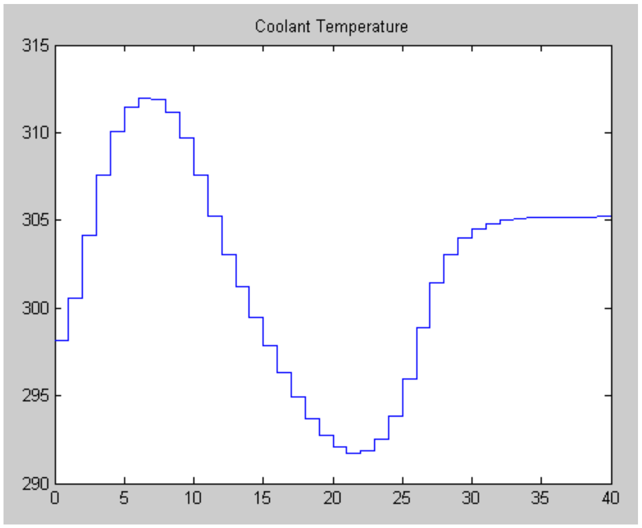
```
figure(2)
plot(tsave,usave(:,3));
title('Coolant Temperature')
```

CSTR Results and Discussion

The plotted results appear below. Note the following points:

- The setpoint is being ramped from the initial concentration to the desired final value (see the step-wise changes in the reactor concentration plot below). The reactor concentration tracks this ramp smoothly with some delay (see the smooth curve), and settles at the final state with negligible overshoot. The controller works equally well (and achieves the final concentration more rapidly) for a step-wise setpoint change, but it makes unrealistically rapid changes in coolant temperature (not shown).
- The final steady state requires a coolant temperature of 305.20 K (see the coolant temperature plot below). An interesting feature of this nonlinear plant is that if one starts at the initial steady state (coolant temperature = 298.15 K), stepping the coolant temperature to 305.20 and holding will not achieve the desired final concentration of 2. In fact, under this simple strategy the reactor concentration stabilizes at a final value of 7.88, far from the desired value. A successful controller must increase the reactor temperature until the reaction “takes off,” after which it must reduce the coolant temperature to handle the increased heat load. The relinearization approach provides such a controller (see following plots).





- Function `linearize` relinearizes the plant as its state evolves. This function was discussed previously in “Linearization Using MATLAB Code”.
- The code also resets the linear model's nominal conditions to the latest values. Note, however, that the first two input signals, which are unmeasured disturbances in the

controller design, always have nominal zero values. As they are unmeasured, the controller cannot be informed of the true values. A non-zero values would cause an error.

- Function `mpc` defines a new controller based on the relinearized plant model. The output weight tuning ignores the temperature measurement, focusing only on the concentration.
- At $t = 0$, the `mpcstate` function initializes the controller's extended state vector, `x`, which is an *mpcstate object*. Thereafter, the `mpcmove` function updates it automatically using the controller's default state estimator. It would also be possible to use an Extended Kalman Filter (EKF) as described in [1] and [2], in which case the EKF would reset the `mpcstate` input variables at each step.
- The `mpcmove` function uses the latest controller definition and state, the measured plant outputs, and the setpoints to calculate the new coolant temperature at each step.
- The Simulink `sim` function simulates the nonlinear plant from the beginning to the end of the control interval. Note that the final condition from the previous step is being used as the initial plant state, and that the plant inputs are being held constant during each interval.

Remember that a conventional feedback controller or a fixed Model Predictive Control Toolbox controller tuned to operate at the initial condition would become unstable as the plant moves to the final condition. Periodic model updating overcomes this problem automatically and provides excellent control under all conditions.

Control Based On Multiple Plant Models

The “Nonlinear CSTR Application” on page 4-9 shows how updates to the prediction model can improve MPC performance. In that case the model is nonlinear and you can obtain frequent updates by linearization.

A more common situation is that you have several linear plant models, each of which applies at a particular operating condition and you can design a controller based on each linear model. If the models cover the entire operating region and you can define a criterion by which you switch from one to another as operating conditions change, the controller set should be able to provide better performance than any individual controller.

The Model Predictive Control Toolbox includes a Simulink block that performs this function. It is the *Multiple MPC Controllers* block. The rest of this section is an illustrative example organized as follows:

- “A Two-Model Plant” on page 4-15
- “Designing the Two Controllers” on page 4-17
- “Simulating Controller Performance” on page 4-18

A Two-Model Plant

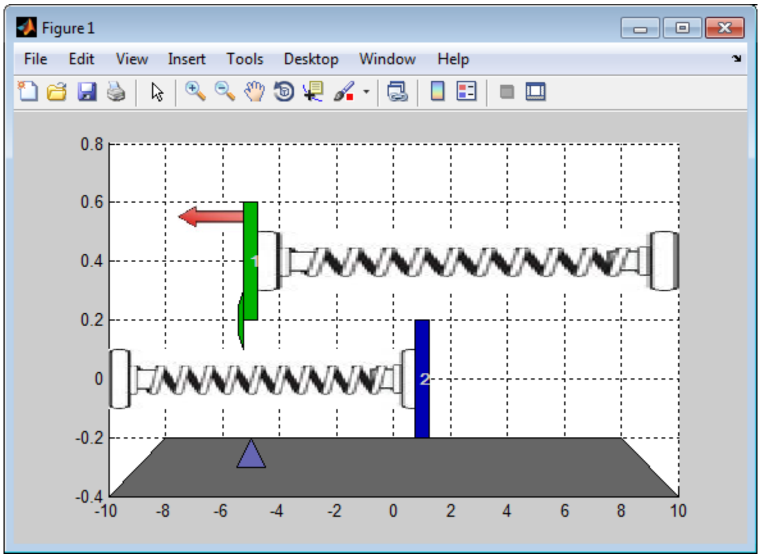
Note Switching MPC Controllers with Multiple MPC Controllers Block provides an animated version of the plant described below.

Animation of the Multi-Model Example is a stop-action snapshot of the subject plant. It consists of two masses, M_1 and M_2. A spring connects M_1 to a rigid wall and pulls it to the right. An applied force, shown as a red arrow in Animation of the Multi-Model Example, opposes this spring, pulling M_1 to the left.

When the two masses are detached, as in Animation of the Multi-Model Example, mass M_2 is uncontrollable and responds only to the spring pulling it to the left.

If the two masses collide, however, they stick together (the collision is completely inelastic) until a change in the applied force separates them.

The control objective is to move M_1 in response to a command signal. The blue triangle in Animation of the Multi-Model Example represents the desired location. At the instant shown, the desired location is –5.



Animation of the Multi-Model Example

In order to achieve its objective, the controller can adjust the applied force magnitude (the length of the red arrow). It receives continuous feedback on the M_1 location. There is also a contact sensor to signal collisions. The M_2 location is unmeasured.

If M_1 were isolated, this would be a routine control problem. The challenge is that the relationship between the applied force and the M_1 movement changes dramatically when M_2 attaches to M_1.

The following code defines the model. First define the system parameters as follows:

```
%% Model Parameters
M1=1;       % mass
M2=5;       % mass
k1=1;       % spring constant
k2=0.1;     % spring constant
b1=0.3;     % friction coefficient
b2=0.8;     % friction coefficient
yeq1=10;    % wall mount position
yeq2=-10;   % wall mount position
```

Next define a model of M_1 when the masses are separated. Its states are the M_1 position and velocity. Its inputs are the applied force, which will be the controller's manipulated variable, and a spring constant calibration signal, which is a measured disturbance input.

```
A1=[0 1;-k1/M1 -b1/M1];
B1=[0 0;-1/M1 k1*yeq1/M1];
C1=[1 0];
D1=[0 0];
sys1=ss(A1,B1,C1,D1);
sys1=setmpcsignals(sys1, 'MV', 1, 'MD', 2);
```

The `setmpcsignals` command specifies the input type for the two inputs.

We need another model (with the same input/output structure) to predict movement when the two masses are joined, as follows:

```
A2=[0 1;-(k1+k2)/(M1+M2) -(b1+b2)/(M1+M2)];
B2=[0 0;-1/(M1+M2) (k1*yeq1+k2*yeq2)/(M1+M2)];
C2=[1 0];
D2=[0 0];
sys2=ss(A2,B2,C2,D2);
sys2=setmpcsignals(sys2, 'MV', 1, 'MD', 2);
```

Designing the Two Controllers

Next we define controllers for each case. Both use a 0.2 second sampling period, a prediction horizon of $P = 20$, a control horizon of $M = 1$, and the default values for all other controller design parameters. The only difference in the controllers is the prediction model.

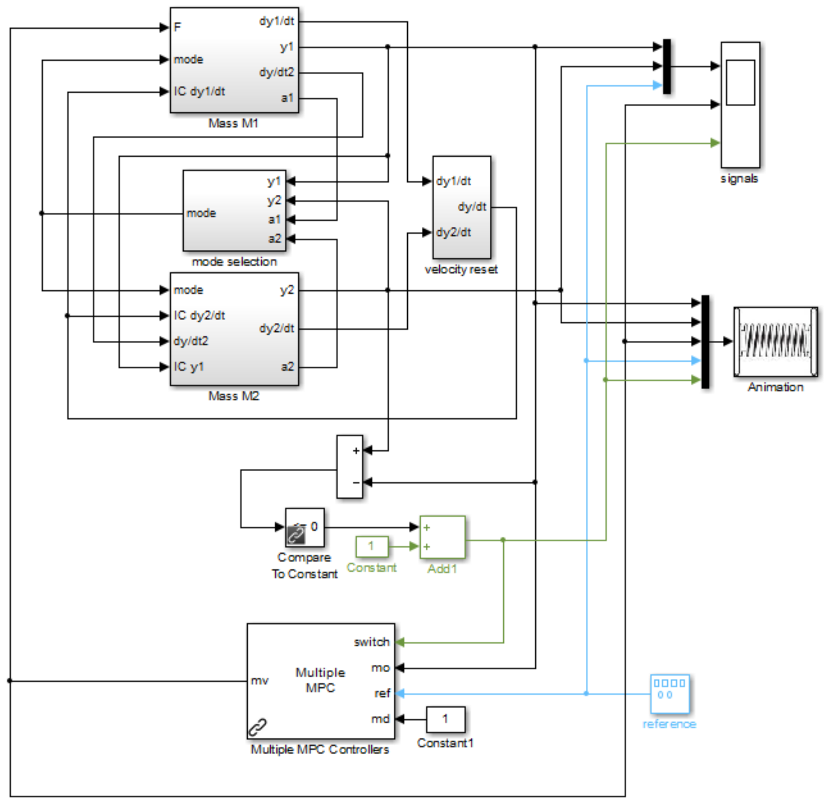
```
Ts=0.2;     % sampling time
p=20;       % prediction horizon
m=1;        % control horizon
MPC1=mpc(sys1,Ts,p,m); % Controller for M1 detached from M2
MPC2=mpc(sys2,Ts,p,m); % Controller for M1 connected to M2
```

The applied force also has the same constraints in each case. Its lower bound is zero (it can't reverse direction), and its maximum rate of change is 1000 per second (increasing or decreasing).

```
MPC1.MV=struct('Min',0,'RateMin',-1e3,'RateMax',1e3);
MPC2.MV=struct('Min',0,'RateMin',-1e3,'RateMax',1e3);
```

Simulating Controller Performance

Block Diagram of the Two-Model Example shows the Simulink block diagram for this example. The upper portion simulates the movement of the two masses, plots the signals as a function of time, and animates the example.



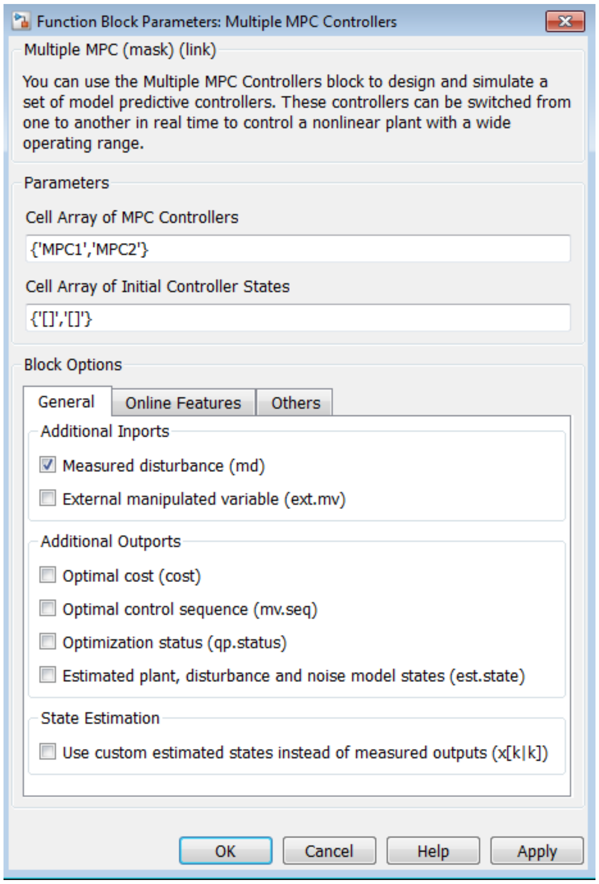
Block Diagram of the Two-Model Example

The lower part contains three key elements:

- A pulse generator that supplies the desired M_1 position (the controller reference signal). Its output is a square wave varying between –5 and 5 with a frequency of 0.015 per second.

- A simulation of a contact sensor. When the two masses have the same position, the `Compare to Constant` block evaluates to `true`, and the `Add1` block converts this to a 2. Otherwise, the `Add1` output is 1.
- The Multiple MPC Controller block. It has four inputs. The measured output (`mo`), reference (`ref`), and measured disturbance (`md`) inputs are as for a standard MPC Controller block. The distinctive feature is the `switch` input.

The figure below shows the Multiple MPC Controller block mask for this example (obtained by double clicking on the controller block).



Multiple MPC (mask) (link)

You can use the Multiple MPC Controllers block to design and simulate a set of model predictive controllers. These controllers can be switched from one to another in real time to control a nonlinear plant with a wide operating range.

Parameters

Cell Array of MPC Controllers

{'MPC1','MPC2'}

Cell Array of Initial Controller States

{'[]','[]'}

Block Options

Additional Inports

[✓] Measured disturbance (md)

[ ] External manipulated variable (ext.mv)

Additional Outports

[ ] Optimal cost (cost)

[ ] Optimal control sequence (mv.seq)

[ ] Optimization status (qp.status)

[ ] Estimated plant, disturbance and noise model states (est.state)

State Estimation

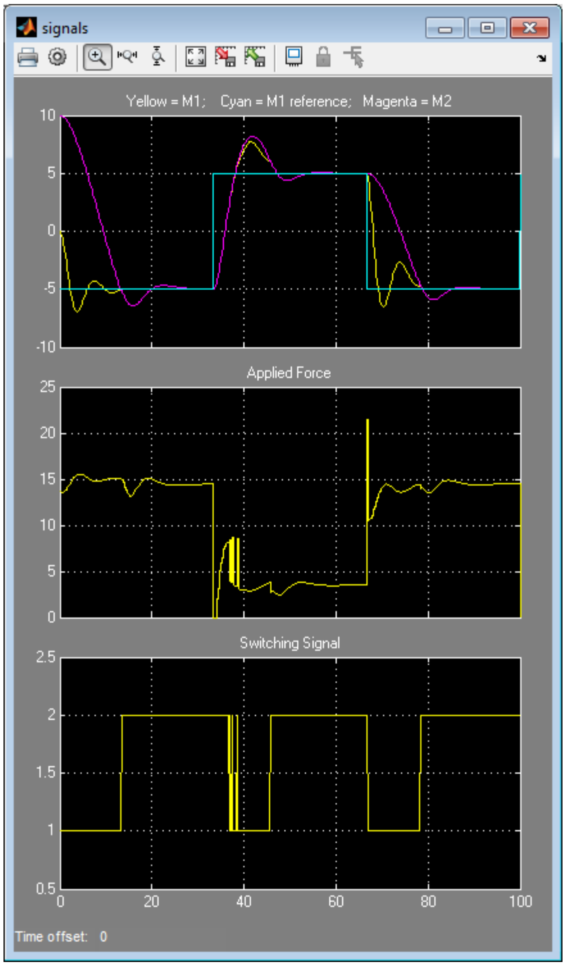
[ ] Use custom estimated states instead of measured outputs (x[k|k])

When the `switch` input is 1 the block automatically activates the first controller listed (`MPC1`), which is appropriate when the masses are separated. When the `switch` input is 2 the block automatically enables the second controller (`MPC2`).

The following code simulates the controller performance

```
Tstop=100;  % Simulation time
y1initial=0;    % Initial M1 and M2 Positions
y2initial=10;
open('mpc_switching');
sim('mpc_switching',Tstop);
```

The figure below shows the `signals` scope output.



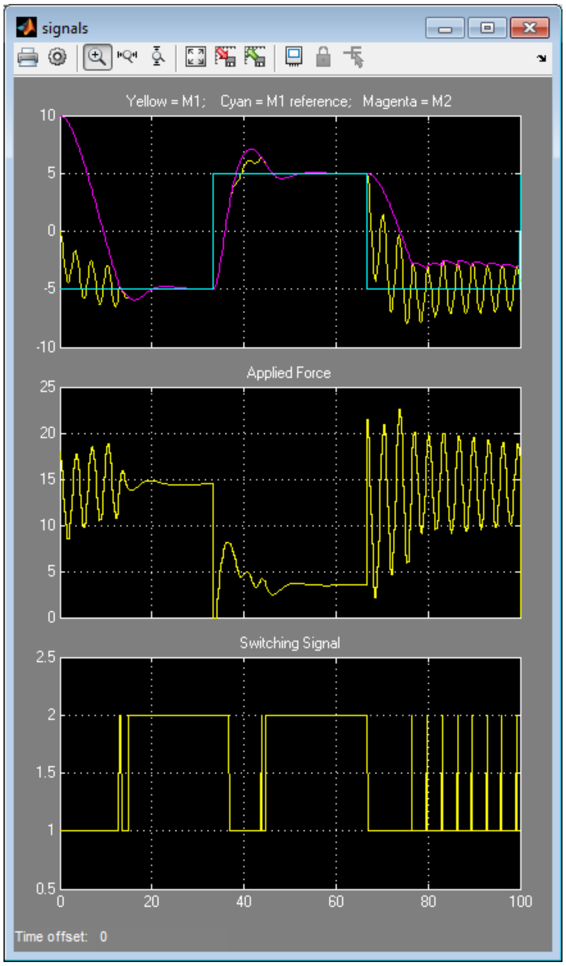
In the upper plot, the cyan curve is the desired position. It starts at –5. The M_1 position (yellow) starts at 0 and under the control of `MPC1`, M_1 moves rapidly toward the desired position. M_2 (magenta) starts at 10 and begins moving in the same direction. At about t = 13 seconds, M_2 collides with M_1. The switching signal (lower plot) changes at this instant from 1 to 2, so controller `MPC2` has taken over.

The collision moves M_1 away from its desired position and M_2 remains joined to M_1. Controller `MPC2` adjusts the applied force (middle plot) so M_1 quickly returns to the desired position.

When the desired position changes step-wise to 5, the two masses separate briefly (with appropriate switching to MPC1) but for the most part move together and settle rapidly at the desired position. The transition back to –5 is equally well behaved.

Now suppose we force MPC2 to operate under all conditions. The figure below shows the result. When the masses are separated, as at the start, MPC2 applies excessive force and then over-compensates, resulting in oscillatory behavior. Once the masses join, the movement smooths out, as would be expected.

The oscillations are especially severe in the last transition. The masses collide frequently and M_1 never reaches the desired position.



If we put `MPC1` in charge exclusively, we instead see sluggish movements that fail to settle at the desired position before the next transition occurs (not shown but you can run `mpcswitching`).

In this case, at least, two controllers are better than one.

Compute Steady-State Gain

This example shows how to analyze a Model Predictive Controller using `cloffset`. The function computes the closed-loop, steady-state gain for each output when subjected to a sustained, 1-unit disturbance added to each output. It assumes that no constraints will be encountered.

For example, consider the controller operating at the final steady-state of the nonlinear `CSTR` of the previous section. To compute its gain, type

```
cloffset(MPCobj)
```

which gives the result:

```
ans =

    1.0000   14.5910
   -0.0000   -0.0000
```

The interpretation is that the controller doesn't react to a sustained disturbance of 1 unit in the first output (the reactor temperature). Recall that we assigned zero weight to this output in the controller design, so the controller ignores deviations from its setpoint. The same disturbance has no effect on the second output (the 2,1 element is zero).

If there is a 1-unit disturbance in the second output, the controller reacts, and the first output increases 14.59 units. This is again due to the zero weight on this output. The second output stays at its setpoint (the 2,2 element is zero).

Extract Controller

This example shows how to obtain an LTI representation of an unconstrained Model Predictive Control Toolbox controller using `ss`. You can use this to analyze the controller's closed-loop frequency response, etc.

For example, consider the controller designed in "Create a Controller Object" on page 4-2. To extract the controller, you could type:

```
MPCss = ss(MPCobj);
```

You could then construct an LTI model of the closed-loop system using the `feedback` function (see Control System Toolbox for details) by typing:

```
CSTRd = c2d(CSTR, MPCss.Ts);
Feedin = 1;
Feedout = 1;
Sign = 1;
CLsys = feedback(CSTRd, MPCss, Feedin, Feedout, Sign);
```

Note The `CSTR` model must be converted to discrete form with the same control interval as the controller.

Recall that the CSTR plant has two inputs and two outputs. The first input is the manipulated variable and the other is an unmeasured disturbance. The first output is measured for feedback and the other is not. The `Feedin` and `Feedout` parameters specify the input and output to be used for control. The `Sign` parameter signifies that the MPC object uses positive feedback, i.e., the measured outputs enter the controller with no sign change. Omission of this would cause the `feedback` command to use negative feedback by default and would almost certainly lead to an unstable closed-loop system.

You could then type

```
eig(CLsys)
```

to verify that all closed-loop poles are within the unit circle, or

```
bode(CLsys)
```

to compute a closed-loop Bode plot.

Bibliography

[1] Lee, J. H. and N. L. Ricker, “Extended Kalman Filter Based Nonlinear Model Predictive Control,” *Ind. Eng. Chem. Res.*, Vol. 33, No. 6, pp. 1530–1541 (1994).

[2] Ricker, N. L., and J. H. Lee “Nonlinear Model Predictive Control of the Tennessee Eastman Challenge Process,” *Computers & Chemical Engineering*, Vol. 19, No. 9, pp. 961–981 (1995).

Signal Previewing

By default, a Model Predictive Controller assumes the current reference and measured plant disturbance signals will remain constant for the duration of the controller's prediction horizon. This emulates conventional feedback control.

As shown in "Optimization Problem", however, the MPC Toolbox optimization problem allows these signals to vary within the prediction horizon. If your application allows you to anticipate trends in such signals, you can use signal previewing in combination with a Model Predictive Controller to improve reference tracking, measured disturbance rejection, or both.

The following Model Predictive Control Toolbox commands provide previewing options:

- `sim`
- `mpcmove`
- `mpcmoveAdaptive`

For Simulink, the following blocks support previewing:

- MPC Controller
- Adaptive MPC Controller
- Multiple MPC Controllers

The Explicit MPC Controller will support previewing in a future release.

Related Examples

- Improving Control Performance with Look-Ahead (Previewing)

More About

- "Run-Time Constraint Updating" on page 4-28

Run-Time Constraint Updating

Constraint bounds can change during controller operation. The `mpcmove`, `mpcmoveAdaptive`, and `mpcmoveExplicit` commands allow for this possibility. In Simulink, all the MPC Controller blocks allow for it as well. Both of these simulation approaches give you the option to update all specified bounds at each control interval. You cannot constrain a variable for which the corresponding controller object property is unbounded.

If your controller object uses time-varying constraints, the updating option applies to the first finite value in the prediction horizon, and all other values adjust to maintain the same profile, i.e., they update by the same amount.

Related Examples

- Varying Input and Output Constraints

More About

- "Run-Time Weight Tuning" on page 4-29

Run-Time Weight Tuning

There are two ways to perform tuning experiments using Model Predictive Control Toolbox software:

- Modify your controller object off line (by changing weights, etc.) and then test the modified object.
- Change tuning weights as the controller operates.

This topic describes the second approach.

You can adjust the following tuning weights as the controller operates (see "Tuning Weights"):

- Plant output variable (OV) reference tracking, w^y.
- Manipulated variable (MV) reference tracking, w^u.
- MV increment suppression, $w^{\Delta u}$.
- Global constraint softening, $\rho_{\#}$.

In each case, the weight applies to the entire prediction horizon. If you are using time-varying weights, you must use offline modification of the weight and then test the modified controller.

In Simulink, the following blocks support online (run-time) tuning:

- MPC Controller
- Adaptive MPC Controller
- Multiple MPC Controllers. In this case, the tuning signals apply to the active controller object, which might switch as the control system operates. If the objects in your set employ different weights, you should tune them off line.

The Explicit MPC Controller block cannot support online tuning because a weight change requires a complete revision of the explicit MPC control law, which is computationally intensive.

For command-line testing, the `mpcmove` and `mpcmoveAdaptive` commands include options to mimic the online tuning behavior available in Simulink.

Related Examples

- Tuning Controller Weights

More About

- "Signal Previewing" on page 4-27

Designing and Testing Controllers in Simulink

The Model Predictive Control Toolbox provides a controller block library for use in Simulink. This allows you to test controller performance in nonlinear simulations. The procedure depends on whether you have already designed a controller. You also have the option to coordinate multiple Model Predictive Controllers.

Design Controller in Simulink

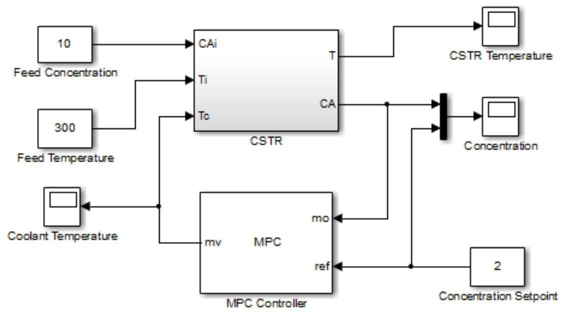
This example shows how to design a model predictive controller in Simulink.

Suppose you have built a nonlinear plant model in Simulink. You want to design an MPC controller at a specific equilibrium operating point. This example shows that model-based design workflow. You linearize the plant at the desired operating point, design the MPC controller and validate it with nonlinear simulation.

Continuous Stirred Tank-Reactor Model

The Simulink model, `CSTR_ClosedLoop`, models a continuous stirred tank-reactor (CSTR) plant. A model predictive controller regulates the CSTR plant.

```
open_system('CSTR_ClosedLoop');
```



CSTR Plant

The CSTR block, which models the nonlinear plant, has three inputs:

- Feed Concentration (CAi inport) — Concentration of the limiting reactant in the CSTR feed stream.
- Feed Temperature (Ti inport) — Temperature of the limiting reactant in the CSTR feed stream.
- Coolant Temperature (Tc inport) — Temperature of the CSTR coolant. The MPC controller (MPC Controller block) treats this plant input as a manipulated variable.

The CSTR block has two states: the temperature and concentration of the limiting reactant in the reactor. These states are measured and displayed by the following blocks:

- CSTR Temperature — Temperature of the limiting reactant in the product stream.
- Concentration — Concentration of the limiting reactant in the product stream, also referred to as *residual concentration*. The MPC controller regulates this state to satisfy the desired setpoint.

MPC Controller

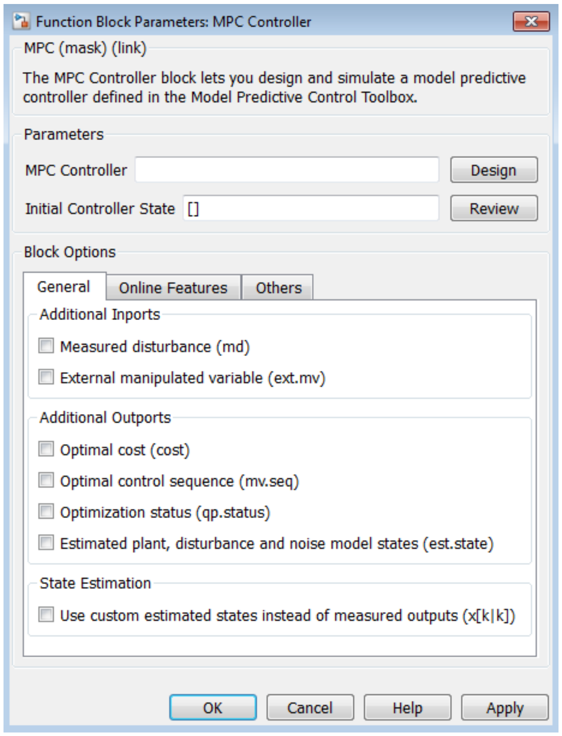
The MPC Controller block, which requires a supported C or C++ compiler, represents the model predictive controller. It is available in the **Model Predictive Control Toolbox** library of the Simulink Library Browser. For this example, the block is configured as follows:

- Measured output (mo inport) — Residual concentration; signal originating at the CA outport of the CSTR block.
- Reference, also referred to as *setpoint* or *target* (ref inport) — Residual concentration reference; signal originating at the Concentration Setpoint block. Set to 2 kmol/m^3.
- Manipulated variable (mv outport) — Concentration of the CSTR coolant; input signal of the Tc inport of the CSTR block.

The reactor temperature is usually controlled. For this example, ignore the reactor temperature for a more challenging nonlinear control problem.

Open MPC Design Tool

Double-click the MPC Controller block to open the block dialog box.



Function Block Parameters: MPC Controller

MPC (mask) (link)

The MPC Controller block lets you design and simulate a model predictive controller defined in the Model Predictive Control Toolbox.

Parameters

MPC Controller: (blank)

Initial Controller State: []

Block Options

Additional Inports

- ☐ Measured disturbance (md)
- ☐ External manipulated variable (ext.mv)

Additional Outports

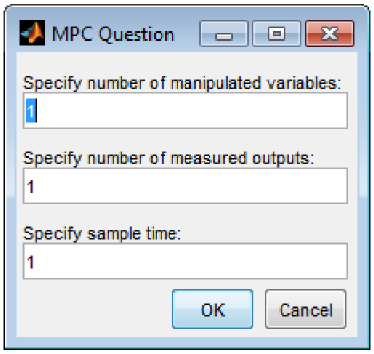
- ☐ Optimal cost (cost)
- ☐ Optimal control sequence (mv.seq)
- ☐ Optimization status (qp.status)
- ☐ Estimated plant, disturbance and noise model states (est.state)

State Estimation

- ☐ Use custom estimated states instead of measured outputs (x[k|k])

The **MPC Controller** box is blank. No controller has been designed yet.

Click **Design**. The MPC Question dialog box opens.



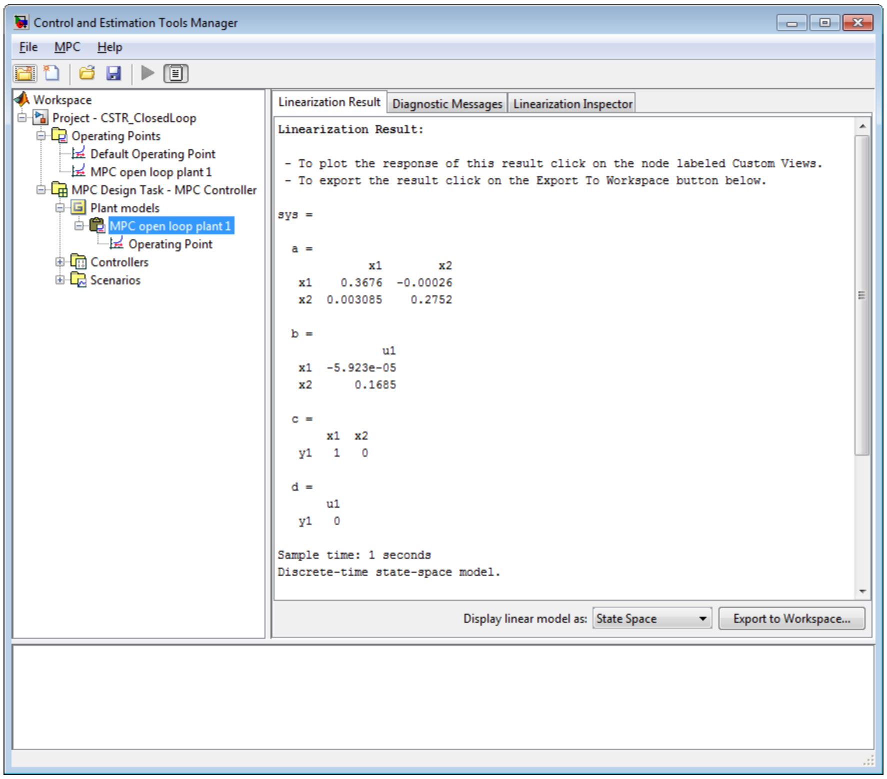
MPC Question

Specify number of manipulated variables: 1

Specify number of measured outputs: 1

Specify sample time: 1

For this example, use the default values of this dialog box. Click **OK**. The software performs multiple tasks to design an initial MPC controller. These tasks include finding an equilibrium operating point, linearizing the plant model, and building the MPC controller. Finally, the Control and Estimation Tools Manager opens, containing an MPC design task node (**MPC Design Task - MPC Controller**). This node includes the linearized plant model, **MPC open loop plant 1**.

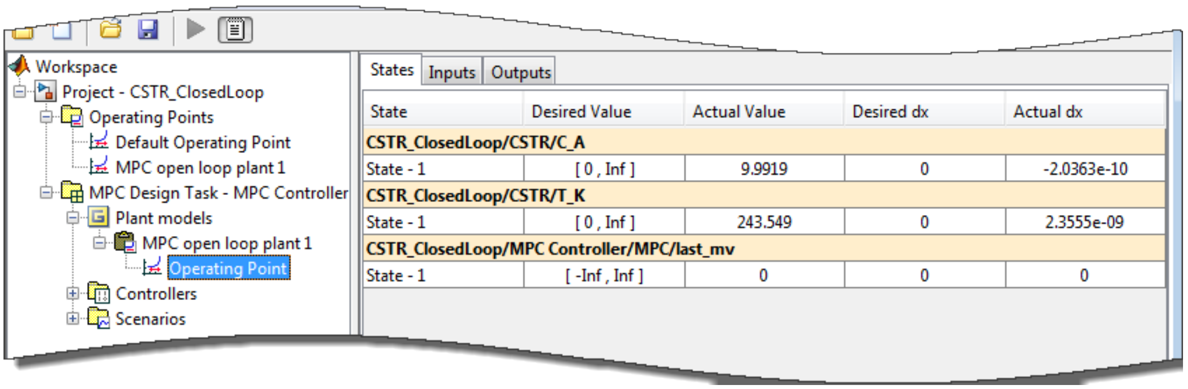


```
Linearization Result:

 - To plot the response of this result click on the node labeled Custom Views.
 - To export the result click on the Export To Workspace button below.

sys =

  a =
              x1        x2
   x1    0.3676  -0.00026
   x2  0.003085    0.2752

  b =
                u1
   x1  -5.923e-05
   x2      0.1685

  c =
       x1  x2
   y1   1   0

  d =
       u1
   y1   0

Sample time: 1 seconds
Discrete-time state-space model.
```

Evaluate Default Operating Point

The CSTR plant model has multiple equilibrium operating points. The software uses the default equilibrium operating point to linearize the plant model. This operating point could differ from the desired operating point. It is important to verify that the software used an appropriate operating point. Otherwise, the linear plant model can be unsuitable for controller design.

In the **MPC open loop plant 1** node of the tree, click **Operating Point**.



State	Desired Value	Actual Value	Desired dx	Actual dx
CSTR_ClosedLoop/CSTR/C_A				
State - 1	[0 , Inf]	9.9919	0	-2.0363e-10
CSTR_ClosedLoop/CSTR/T_K				
State - 1	[0 , Inf]	243.549	0	2.3555e-09
CSTR_ClosedLoop/MPC Controller/MPC/last_mv				
State - 1	[-Inf , Inf]	0	0	0

The software found a steady-state operating point (derivatives are close to zero). However, the reactor temperature (**CSTR_ClosedLoop/CSTR/T_K**) is very low, yielding a low reaction rate. Compare the values of the residual concentration state (9.9919 kmol/m^3) and the feed concentration (10 kmol/m^3) to confirm the low reaction rate.

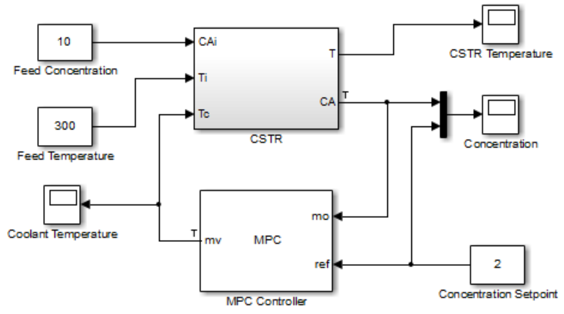
Because this operating point does not result in a significant reaction, calculate another operating point with a higher reaction rate.

Calculate Desired Operating Point for Plant Model

Suppose the desired operating point is:

- Residual concentration is 2 kmol/m^3
- Reactor temperature is 370 K

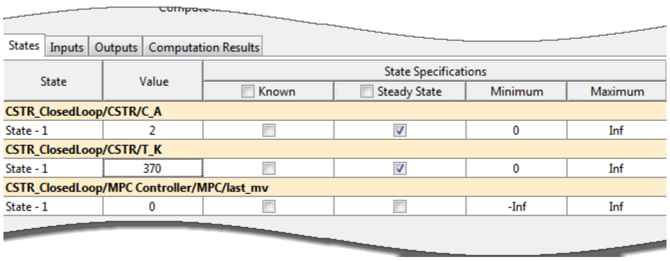
Specify output constraints on all the OV/MV/MD signals connected to the MPC Controller. This step is required because the software uses the steady-state values of these signals as the nominal values for the controller. In the Simulink model, right-click the MO signal entering the controller. Select **Linear Analysis Points> Trim Output Constraint**. Repeat for the MV signal of the MPC Controller block. All constrained signals are marked with **T**.



In the tree of the Control and Estimation Tools Manager, click **Operating Points**. Then, click the **Compute Operating Points** tab.

Specify the desired operating point states:

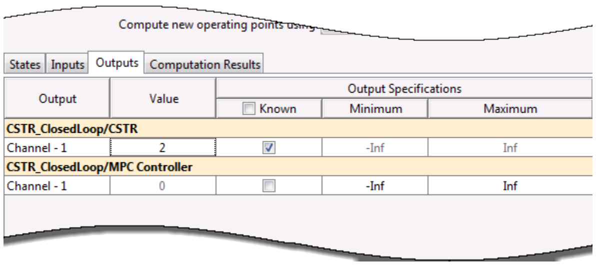
- **CSTR_ClosedLoop/CSTR/C_A** (residual concentration) — Enter 2 in the **Value** box.
- **CSTR_ClosedLoop/CSTR/T_K** (reactor temperature) — Enter 370 in the **Value** box.
- **CSTR_ClosedLoop/MPC Controller/MPC/last_mv** — Clear the **Steady State** check box. This state, which belongs to the MPC Controller block, can be arbitrary. Its value is set by the software during the operating point calculation.



State	Value	State Specifications			
		Known	Steady State	Minimum	Maximum
CSTR_ClosedLoop/CSTR/C_A					
State - 1	2	☐	✓	0	Inf
CSTR_ClosedLoop/CSTR/T_K					
State - 1	370	☐	✓	0	Inf
CSTR_ClosedLoop/MPC Controller/MPC/last_mv					
State - 1	0	☐	☐	-Inf	Inf

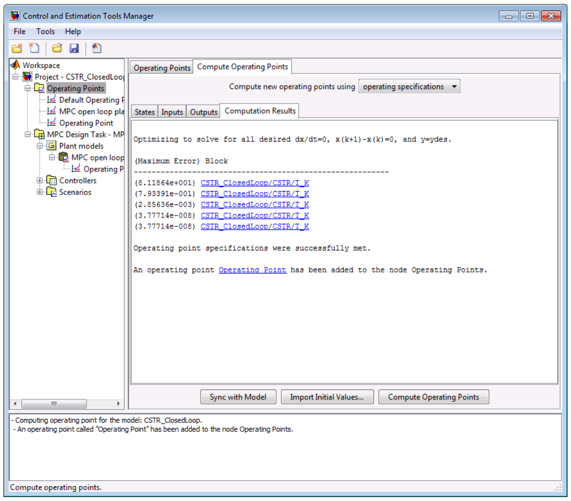
Specify the desired operating point outputs. Click the **Outputs** tab.

- **CSTR_ClosedLoop/CSTR** (reactor concentration) — Enter 2 in the **Value** box. Then, select **Known** check box.
- **CSTR_ClosedLoop/MPC Controller** (coolant temperature) — Use default values.



Output	Value	Output Specifications		
		Known	Minimum	Maximum
CSTR_ClosedLoop/CSTR				
Channel - 1	2	✓	-Inf	Inf
CSTR_ClosedLoop/MPC Controller				
Channel - 1	0	☐	-Inf	Inf

Click **Compute Operating Points**.



```
Optimizing to solve for all desired dx/dt=0, x(k+1)-x(k)=0, and y=ydes.

(Maximum Error) Block
---------------------------------------------------------
(8.11864e+001) CSTR_ClosedLoop/CSTR/T_K
(7.93391e-001) CSTR_ClosedLoop/CSTR/T_K
(2.85636e-003) CSTR_ClosedLoop/CSTR/T_K
(3.77714e-008) CSTR_ClosedLoop/CSTR/T_K
(3.77714e-008) CSTR_ClosedLoop/CSTR/T_K

Operating point specifications were successfully met.

An operating point Operating Point has been added to the node Operating Points.
```

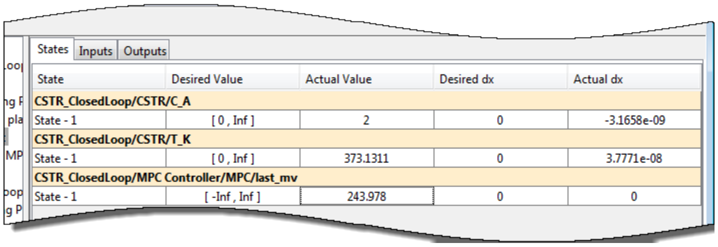
```
- Computing operating point for the model: CSTR_ClosedLoop.
- An operating point called "Operating Point" has been added to the node Operating Points.
```

The **Computation Results** tab displays the operating point search result. The new operating point, `Operating Point`, appears in the **Operating Points** node.

Tip If an operating point computation fails, you can:

- Specify better initial estimates of the unknown quantities.
- Select a different optimization algorithm for the operating point search. To specify a different optimization algorithm, in the Controls and Estimation Tools Manager, select **Options** in the **Tools** menu. Specify the algorithm options in the **Operating Point Search** tab.

Confirm that `Operating Point` is the desired operating point. In the tree, click **Operating Point** in the **Operating Points** node.



States | Inputs | Outputs

State	Desired Value	Actual Value	Desired dx	Actual dx
CSTR_ClosedLoop/CSTR/C_A				
State - 1	[0 , Inf]	2	0	-3.1658e-09
CSTR_ClosedLoop/CSTR/T_K				
State - 1	[0 , Inf]	373.1311	0	3.7771e-08
CSTR_ClosedLoop/MPC Controller/MPC/last_mv				
State - 1	[-Inf , Inf]	243.978	0	0

As desired, the reactor temperature is close to 370 K at steady state. Similarly, the reactor concentration is 2 kmol/m^3 at steady state. `Operating Point` is verified as the desired operating point.

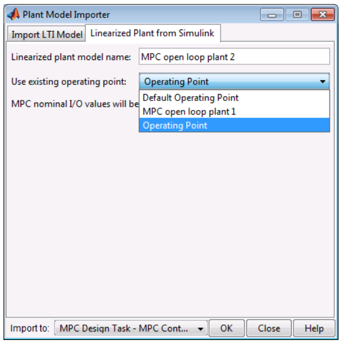
Linearize Plant at Desired Operating Point

In the tree, click **MPC Design Task - MPC Controller**.

Click **Import Plant**. The Plant Model Importer dialog box opens.

Click the **Linearized Plant from Simulink** tab.

- **Linearized plant model name** box (name of the linearized plant to be created) — Use the default name, `MPC open loop plant2`.
- **Use existing operating point** list — Select **Operating Point**.



Plant Model Importer

Import LTI Model | Linearized Plant from Simulink

Linearized plant model name: MPC open loop plant 2

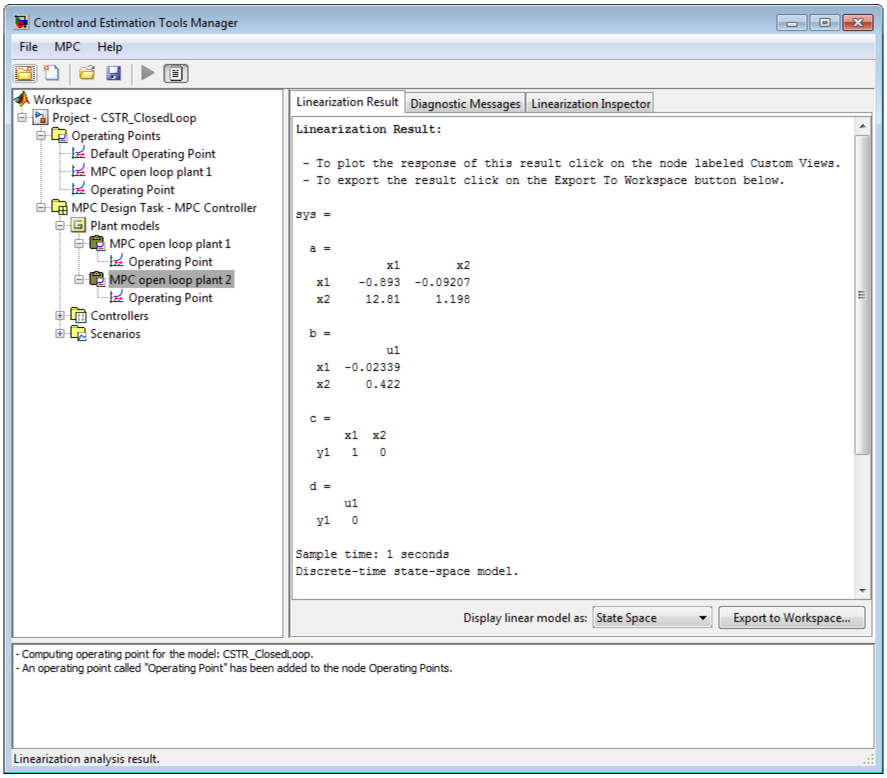
Use existing operating point: Operating Point

MPC nominal I/O values will be

Default Operating Point
MPC open loop plant 1
Operating Point

Import to: MPC Design Task - MPC Cont... | OK | Close | Help

Click **OK**. The software calculates the linearized plant model, **MPC open loop plant2**. This model is visible in the Control and Estimation Tools Manager, in the **Plant models** node.

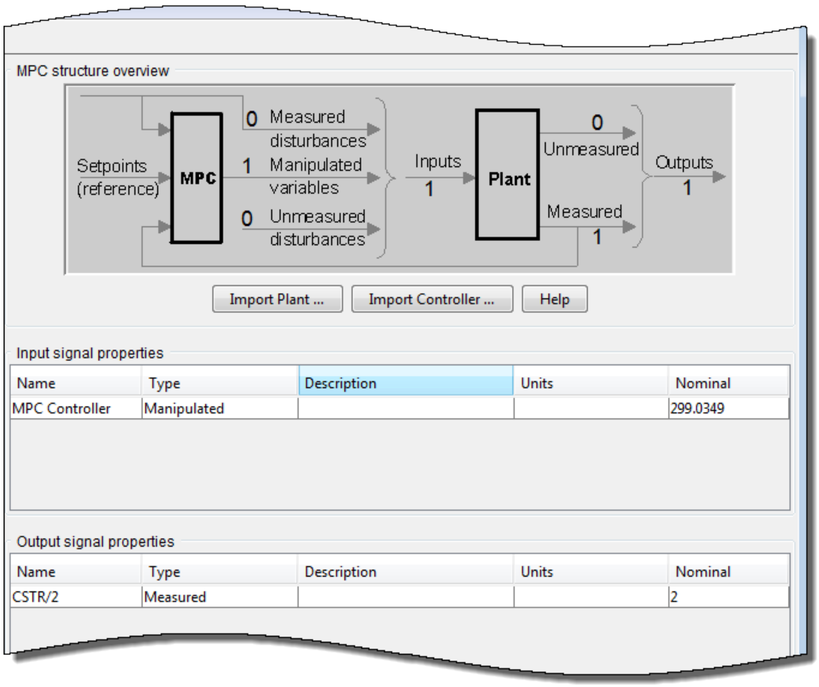


```
Linearization Result:

 - To plot the response of this result click on the node labeled Custom Views.
 - To export the result click on the Export To Workspace button below.

sys =

  a =
              x1        x2
   x1     -0.893  -0.09207
   x2      12.81     1.198

  b =
              u1
   x1   -0.02339
   x2      0.422

  c =
        x1  x2
   y1    1   0

  d =
        u1
   y1    0

Sample time: 1 seconds
Discrete-time state-space model.
```

```
- Computing operating point for the model: CSTR_ClosedLoop.
- An operating point called "Operating Point" has been added to the node Operating Points.
```

(Optional) Close the Plant Model Importer dialog box by clicking **Close**.

(Optional) To see the nominal values of the input and output signals, in the tree, click **MPC Design Task - MPC Controller**.



Input signal properties

Name	Type	Description	Units	Nominal
MPC Controller	Manipulated			299.0349

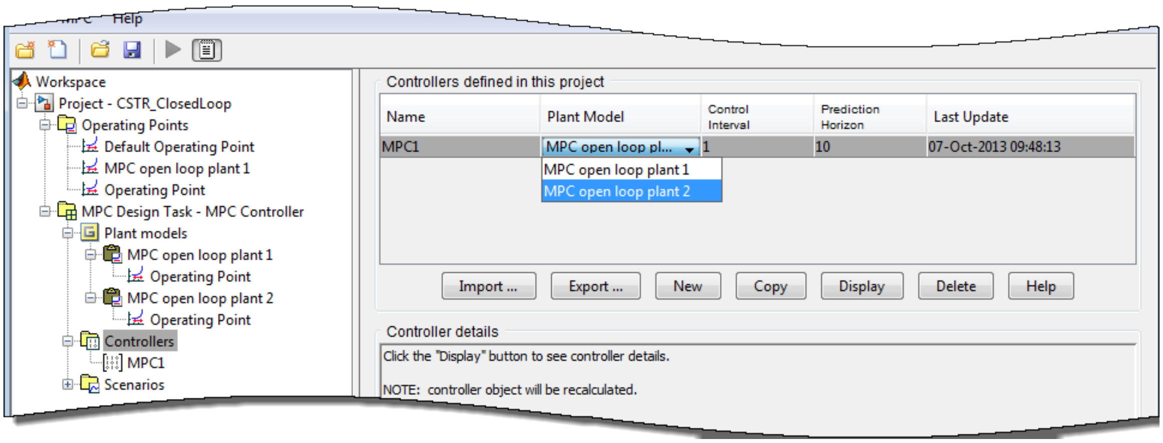
Output signal properties

Name	Type	Description	Units	Nominal
CSTR/2	Measured			2

These values correspond to the desired operating point.

Update Controller to Use New Plant Model

In the tree, click **Controllers**. Then, in the **Plant Model** list, select `MPC open loop plant2`.



Name	Plant Model	Control Interval	Prediction Horizon	Last Update
MPC1	MPC open loop pl...	1	10	07-Oct-2013 09:48:13

MPC open loop plant 1
MPC open loop plant 2

Click the "Display" button to see controller details.

NOTE: controller object will be recalculated.

Alternatively, delete **MPC open loop plant1**, the model computed at default operating point. Deleting this plant forces the controller to use the new model and eliminates the potential for confusion. In the tree, click **Plant models**. Then, in the **Plant models imported for this project** list box, select `MPC open loop plant 1`. Click **Delete**.

Generally, the next step is to configure the controller, including specifying the control and prediction horizons, constraints, weights, and noise/disturbance models. For such configuration, select the controller in the **Controllers** node. However, for this example, use the default controller settings.

(Optional) You can validate the controller design by performing linear simulations. To do so, use the **Scenarios** node.

Export Controller and Operating Point to MATLAB Workspace

By default, the software creates a controller, **MPC1**, based on the model obtained by automatic linearization. The software automatically exports **MPC1** to the MATLAB workspace. Also, in the Simulink model, the software updates the MPC Controller block. It sets the **MPC Controller** parameter to `MPC1`. So, when you run the Simulink model, you use **MPC1**.

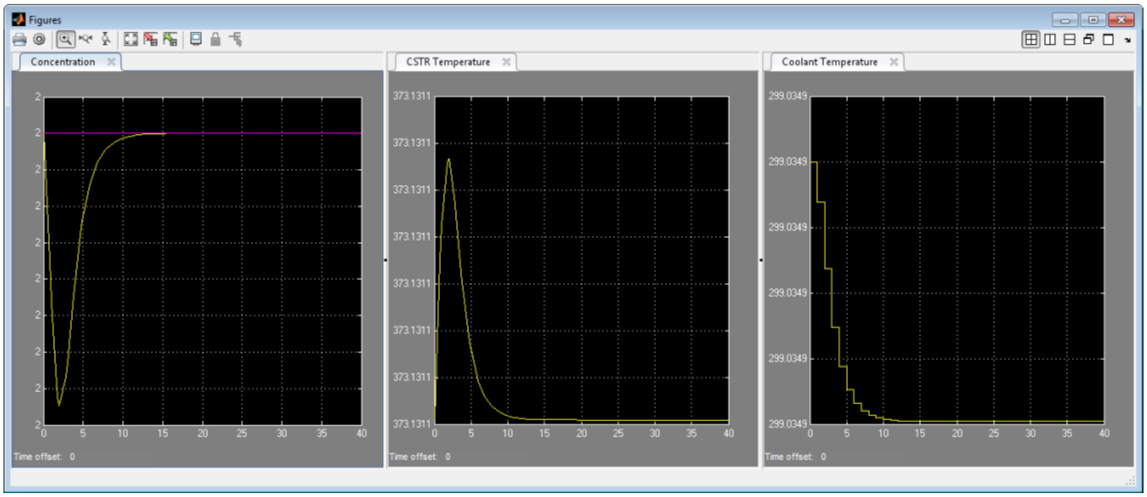
Because the initial linearization was unsuitable, you must export the controller with the new plant model. In the **Controllers** node, right-click **MPC1** and select **Export Controller**. The MPC Controller Exporter dialog box opens. Use the default values specified in this dialog box. Click **Export** and confirm that you want to replace the existing `MPC1` in the MATLAB workspace. Optionally, close this dialog box by clicking **Close**.

Tip After you export the controller, you can examine it for design errors and stability problems using `review`.

Verify Steady-State Operation for Plant-Controller Combination

To simulate the model at the desired operating point, export `Operating Point` to the MATLAB workspace and initialize the model with it. In the Control and Estimation Tools Manager, expand the **Operating Points** node and right-click **Operating Point**. Select **Export to Workspace**. The Export to Workspace dialog box opens. Select the **Use the operating point to initialize model** check box. Click **OK**.

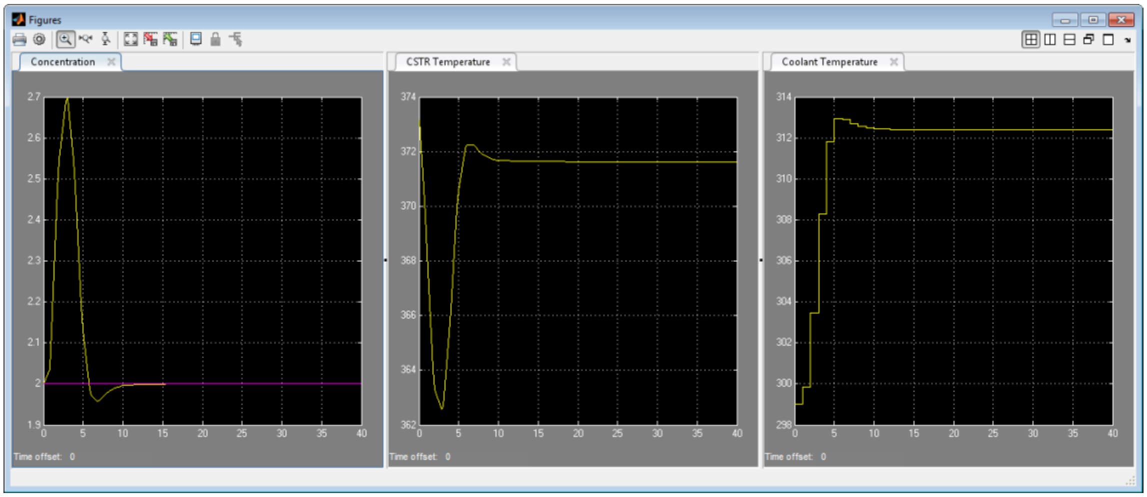
Run the `CSTR_ClosedLoop` model.



The simulation plots show that the residual concentration reaches 2 kmol/m^3 and the reactor temperature is approximately 373 K. Therefore, the controller is initialized at the desired operating point.

Test Controller Robustness

For example, change the feed concentration, specified by the Feed Concentration block, to 9.5 kmol/m^3. Recall that the nominal feed concentration value used to design the controller is 10 kmol/m^3. Run the model.



The decrease in feed concentration reduces heat generation. If the controller were absent, the reactor temperature would drop significantly, reducing the reaction rate, and the CSTR concentration would increase to roughly 3 kmol/m^3. To counteract these effects, the controller raises the coolant temperature, which returns the CSTR temperature to a value slightly below the nominal condition.

See Also

`review`

Related Examples

- "Linearize Simulink Models"
- "Design Controller Using the Design Tool"

Test an Existing Controller

If you have already designed a model predictive controller to use with a Simulink plant, to test it in Simulink:

1. Open Simulink model.
2. Add an MPC Controller block to the model and connect it.
3. Specify the controller.

   Open the MPC Controller and enter the name of your controller in the **MPC Controller** box. The string you enter here must be the name of an `mpc` object in the workspace.
4. (Optional) Modify the controller.

   To use `mpctool`, click **Design** with the MPC Controller open. Change the controller parameters and then export the modified controller to the workspace.

   Alternatively, use commands to modify the controller object in the workspace.
5. Run the Simulink model.

Schedule Controllers at Multiple Operating Points

In this section...

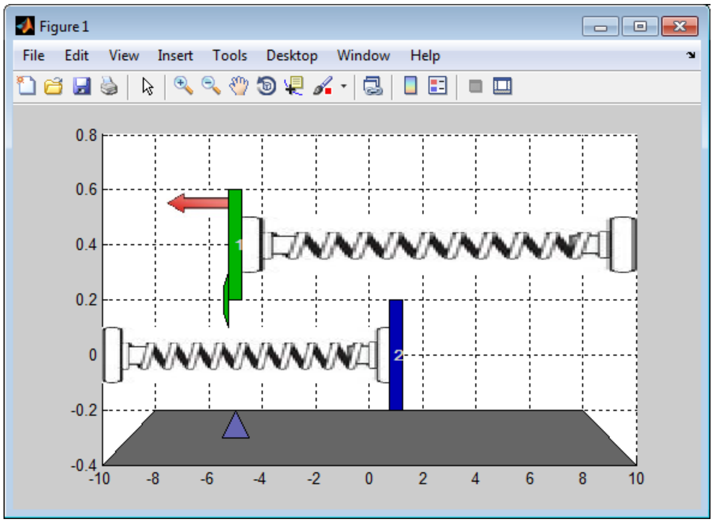
If your plant is nonlinear, a controller designed to operate in a particular target region may perform poorly in other regions. The common way to compensate is to design multiple controllers, each intended for a particular combination of operating conditions. Switch between them in real-time as conditions change. Gain scheduling is a conventional example of this technique. The following example shows how to coordinate multiple model predictive controllers for this purpose.

The rest of this section is an illustrative example organized as follows:

- "A Two-Model Plant" on page 5-16 describes the example application involving two distinct operating conditions.
- "Animation of the Multi-Model Example" on page 5-17 explains how to simulate the plant.
- "Designing the Two Controllers" on page 5-18 shows how to obtain the controllers for each operating condition.
- "Simulating Controller Performance" on page 5-19 shows how to use the Multiple MPC Controllers block in Simulink to coordinate the two controllers.

A Two-Model Plant

The figure below is a stop-action snapshot of the plant. It consists of two masses, `M1` (upper) and `M2` (lower). A spring connects `M1` to a rigid wall and pulls it to the right. An applied force, shown as a red arrow, opposes the spring pulling `M1` to the left.



In the above, mass M2 is uncontrollable. It responds solely to the spring pulling it to the left.

If the two masses collide, however, they stick together until a change in the applied force separates them.

The control objective is to make M1 track a reference signal. The blue triangle in the above indicates the desired location, which varies with time. At the instant shown, the desired numeric location is -5.

For an animated version of the plant, see Scheduling Controllers for a Plant with Multiple Operating Points.

Animation of the Multi-Model Example

In order to achieve its objective, the controller can adjust the applied force magnitude (the length of the red arrow). It receives continuous measurements of the M1 location. There is also a contact sensor to signal collisions. The M2 location is unmeasured.

If M1 were isolated, this would be a routine control problem. The challenge is that the relationship between the applied force and M1's movement changes dramatically when M2 attaches to M1.

Define the model.

Define the system parameters.

```
M1 = 1; % mass
M2 = 5; % mass
k1 = 1; % spring constant
k2 = 0.1; % spring constant
b1 = 0.3; % friction coefficient
b2 = 0.8; % friction coefficient
yeq1 = 10; % wall mount position
yeq2 = -10; % wall mount position
```

Define a model of `M1` when the masses are separated. Its states are the position and velocity of `M1`. Its inputs are the applied force, which will be the controller's manipulated variable (MV), and a spring constant calibration signal, which a measured disturbance (MD) input.

```
A1 = [0 1;-k1/M1 -b1/M1];
B1 = [0 0;-1/M1 k1*yeq1/M1];
C1 = [1 0];
D1 = [0 0];
sys1 = ss(A1,B1,C1,D1);
sys1 = setmpcsignals(sys1,'MV',1,'MD',2);
```

The call to `setmpcsignals` specifies the input type for the two inputs.

Define another model (with the same input/output structure) to predict movement when the two masses are joined.

```
A2 = [0 1;-(k1+k2)/(M1+M2) -(b1+b2)/(M1+M2)];
B2 = [0 0;-1/(M1+M2) (k1*yeq1+k2*yeq2)/(M1+M2)];
C2 = [1 0];
D2 = [0 0];
sys2 = ss(A2,B2,C2,D2);
sys2 = setmpcsignals(sys2,'MV',1,'MD',2);
```

Designing the Two Controllers

Next, define controllers for each case. Both controllers employ a 0.2 second sampling period (`Ts`), a prediction horizon of `p = 20`, a control horizon of `m = 1` and default values for all other controller parameters.

The only property that distinguishes the two controllers is the prediction model.

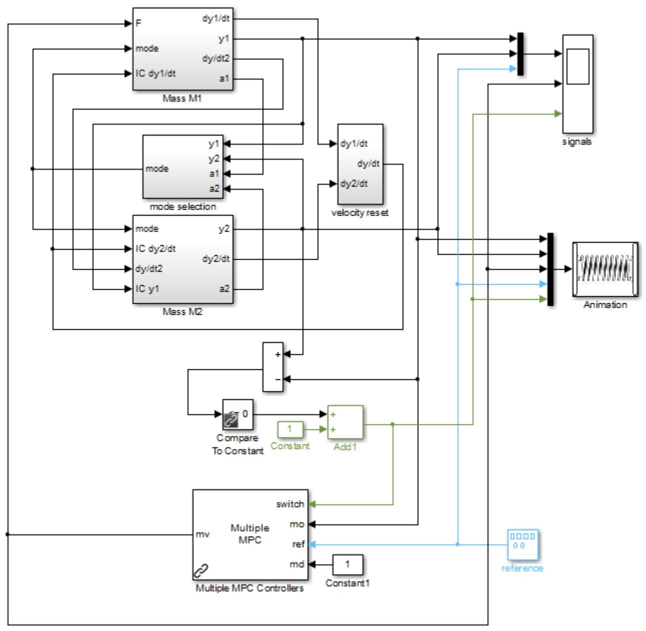
```
Ts = 0.2;
p = 20;
m = 1;
MPC1 = mpc(sys1,Ts,p,m); % Controller for M1 detached from M2
MPC2 = mpc(sys2,Ts,p,m); % Controller for M1 connected to M2
```

The applied force also have the same constraints in each case. The lower bound for the applied force is zero because it cannot reverse direction. The maximum rate of change for the applied force is 1000 per second (increasing or decreasing).

```
MPC1.MV = struct('Min',0,'RateMin',-1e3,'RateMax',1e3);
MPC2.MV = struct('Min',0,'RateMin',-1e3,'RateMax',1e3);
```

Simulating Controller Performance

The following is the Simulink block diagram for this example.

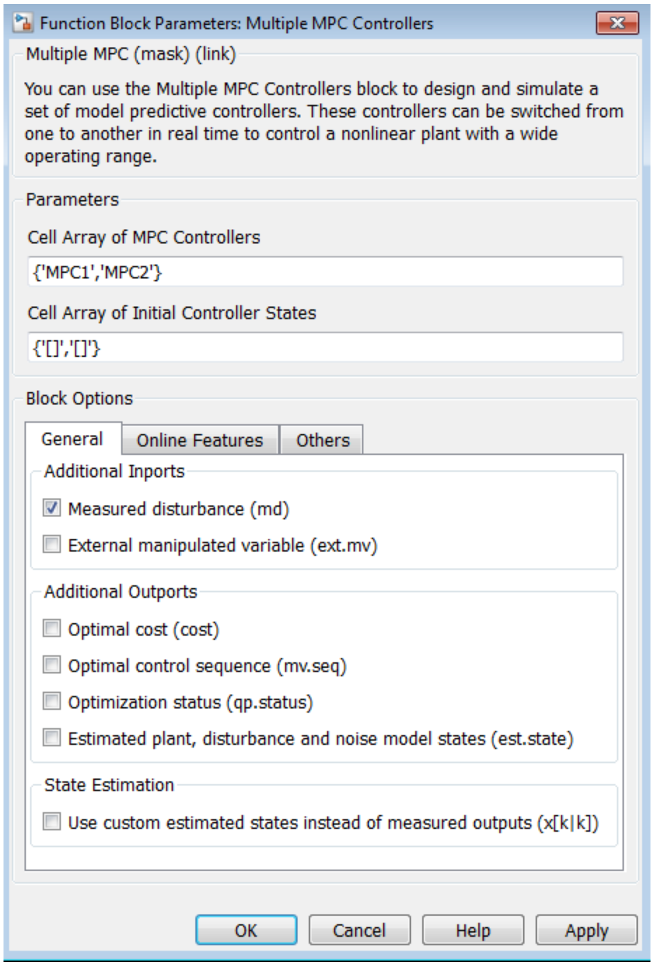


The upper portion simulates the movement of the two masses, plots the signals as a function of time and animates the plant.

The lower part contains the following key elements:

- A pulse generator that supplies the desired `M1` position (the controller reference signal). The output of the pulse generator is a square wave that varies between —5 and 5, with a frequency of 0.015 per second.
- Simulation of a contact sensor. When the two masses have the same position, the Compare to Constant block evaluates to `true` and the `Add1` block converts this to a 2. Else, the output of `Add1` is 1.
- The Multiple MPC Controllers block, which has four inputs. The measured output (`mo`), reference (`ref`) and measured disturbance (`md`) inputs are as for the MPC Controller block. The distinctive feature of the Multiple MPC Controllers block is the `switch` input.

  The figure below shows the Multiple MPC Controllers block mask for this example.


Function Block Parameters: Multiple MPC Controllers

Multiple MPC (mask) (link)

You can use the Multiple MPC Controllers block to design and simulate a set of model predictive controllers. These controllers can be switched from one to another in real time to control a nonlinear plant with a wide operating range.

Parameters

Cell Array of MPC Controllers

{'MPC1','MPC2'}

Cell Array of Initial Controller States

{'[]','[]'}

Block Options

General | Online Features | Others

Additional Inports

- [✓] Measured disturbance (md)
- [ ] External manipulated variable (ext.mv)

Additional Outports

- [ ] Optimal cost (cost)
- [ ] Optimal control sequence (mv.seq)
- [ ] Optimization status (qp.status)
- [ ] Estimated plant, disturbance and noise model states (est.state)

State Estimation

- [ ] Use custom estimated states instead of measured outputs (x[k|k])

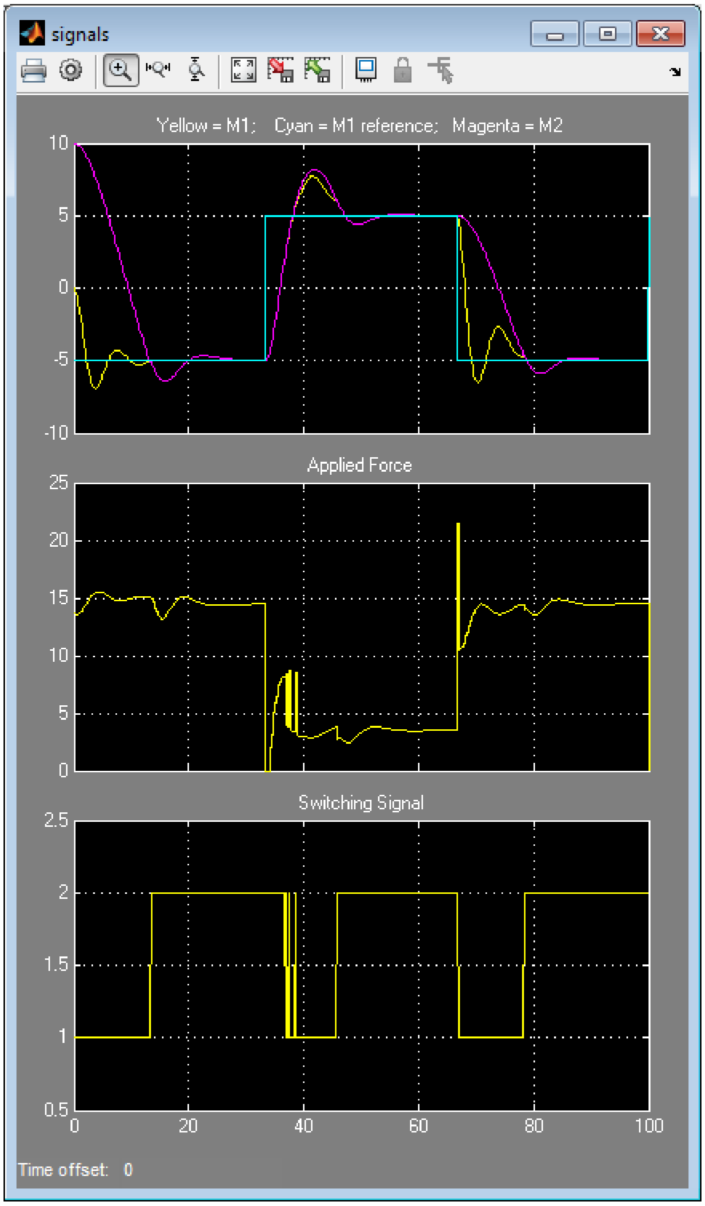
When the `switch` input is 1, the block automatically activates the first controller that is listed (`MPC1`), which is appropriate when the masses are separated. When the `switch` input is 2, the block automatically enables the second controller (`MPC2`).

Simulate the controller performance using Simulink commands.

```
Tstop = 100; % Simulation time
y1initial = 0; % Initial M1 and M2 Positions
```

```
y2initial = 10;
open('mpc_switching');
sim('mpc_switching',Tstop);
```

The figure below shows the `signals` scope output.


signals
Yellow = M1; Cyan = M1 reference; Magenta = M2
10
5
0
-5
-10
Applied Force
25
20
15
10
5
0
Switching Signal
2.5
2
1.5
1
0.5
0
20
40
60
80
100
Time offset: 0

In the upper plot, the cyan curve is the desired position. It starts at -5. The M1 position (yellow) starts at 0. Under the control of MPC1, M1 moves rapidly towards the desired position. M2 (magenta) starts at 10 and begins moving in the same direction.

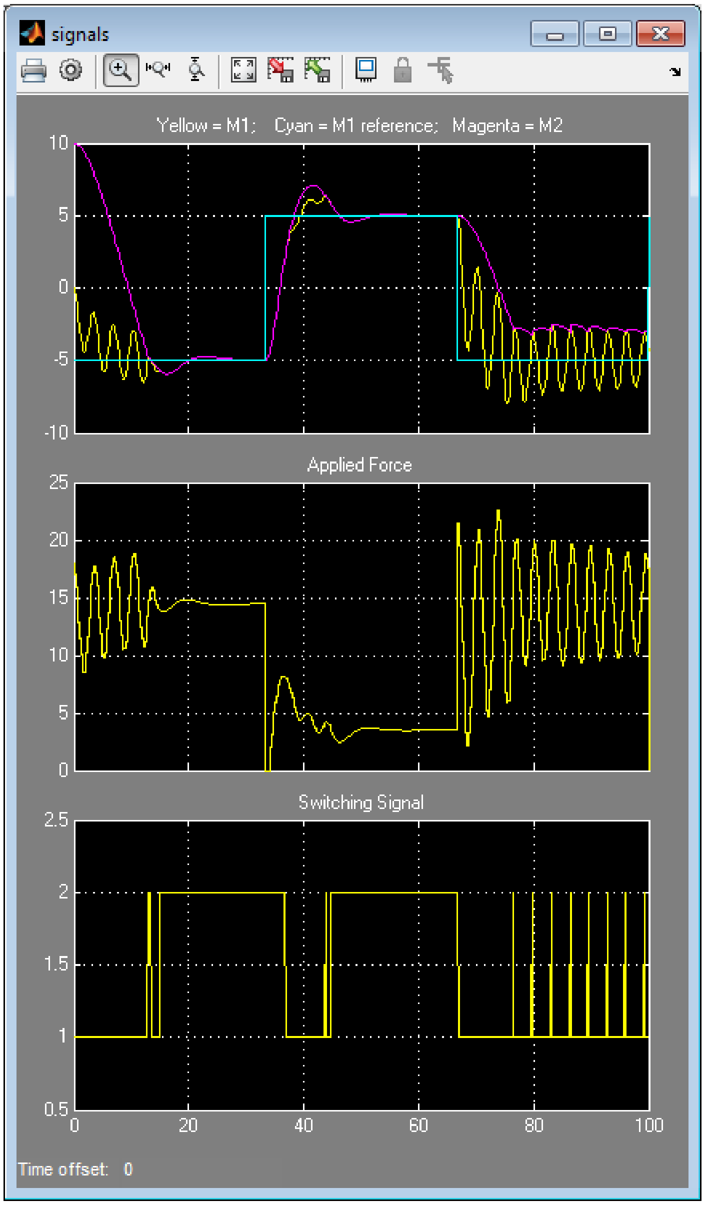
At about t = 13 seconds, M2 collides with M1. The switching signal (lower plot) changes at this instant from 1 to 2, so MPC2 takes over.

The completely inelastic collision moves M1 away from its desired position and M2 remains joined to M1. Controller MPC2 adjusts the applied force (middle plot) so M1 quickly returns to the desired position.

When the desired position changes step-wise to 5, the two masses separate briefly, with the controller switching to MPC1 appropriately. But, for the most part, the two masses move together and settle rapidly at the desired location. The transition back to —5 is equally well behaved.

Suppose we force MPC2 to operate under all conditions.

The figure below shows the result.


signals
Yellow = M1; Cyan = M1 reference; Magenta = M2
10
5
0
-5
-10
Applied Force
25
20
15
10
5
0
Switching Signal
2.5
2
1.5
1
0.5
0
20
40
60
80
100
Time offset: 0

When the masses are disconnected, as at the start, MPC2 applies excessive force and then over-compensates, resulting in oscillatory behavior. Once the masses join, the move more smoothly, as would be expected.

The last transition causes especially severe oscillations. The masses collide frequently and M1 never reaches the desired position.

If MPC1 is in charge exclusively, the masses move sluggishly and fail to settle at the desired position before the next transition occurs. For more information, see the Scheduling Controllers for a Plant with Multiple Operating Points.

In this application, at least, two controllers are, indeed, better than one.